Name: Date:
Teacher: Class/Period:

1) What is AB ?

$$A = \begin{bmatrix} -3 & 1 \\ 6 & 0 \\ 4 & 2 \\ 9 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 6 \\ 5 & 1 \end{bmatrix}$$

A. $\begin{bmatrix} 52 & 156 \\ 130 & 26 \end{bmatrix}$

B. $\begin{bmatrix} -1 & -17 \\ 12 & 36 \\ 18 & 26 \\ 53 & 61 \end{bmatrix}$

C. $\begin{bmatrix} -6 & 6 \\ 30 & 0 \\ 8 & 12 \\ 45 & 7 \end{bmatrix}$

D. $\begin{bmatrix} -42 & 14 \\ 84 & 0 \\ 56 & 28 \\ 126 & 98 \end{bmatrix}$

2) Solve for x.

$$\begin{bmatrix} 25 & -8 \\ 4x & 0 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ x & 3 \end{bmatrix} \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ 6 & x^2 \end{bmatrix}$$

A. $-9, 3$

B. $\dfrac{-1 \pm \sqrt{37}}{2}$

C. $\dfrac{5 \pm i\sqrt{11}}{2}$

D. $\dfrac{1 \pm i\sqrt{35}}{2}$

3) If A is a 3×2 matrix, B is a 3×3 matrix, and C is a 2×3 matrix, what are the dimensions of $A \times C \times B$?

A. 3×3

B. 2×2

C. 2×3

D. 18×18

4) Find the values of x and y for this matrix equation:

$$\begin{bmatrix} x & 5 & 7 \\ 2 & 8 & 4 \end{bmatrix} \bullet \begin{bmatrix} 1 & 3 \\ y & 2 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 68 & 61 \\ 70 & 46 \end{bmatrix}$$

A. $x = 19$, $y = -2$

B. $x = 18$, $y = 2$

C. $x = 3$, $y = 6$

D. $x = -2$, $y = 5$

5) Given $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 0 & -3 \end{bmatrix}$, what is AB ?

A. $\begin{bmatrix} -3 \end{bmatrix}$

B. $\begin{bmatrix} 6 \\ 0 \\ 9 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 6 \\ 2 & 0 \\ 3 & -3 \end{bmatrix}$

D. $\begin{bmatrix} 6 & 0 & -3 \\ 12 & 0 & -6 \\ 18 & 0 & -9 \end{bmatrix}$

6) Determine the sum $\begin{bmatrix} 3 & 0 & 0 & -1 \\ 8 & -2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -3 & -3 & -3 \end{bmatrix}$, if it exists.

A. $\begin{bmatrix} 4 & 1 & 1 & 0 \\ 5 & -5 & -3 & -3 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 1 & 1 & 0 \\ 11 & 1 & 3 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 6 & 6 \\ 14 & 14 \end{bmatrix}$

D. The sum does not exist.

7) Four teams participate in a math competition. The number of 1st, 2nd, 3rd, and 4th place finishes in each round determines the final score. This matrix shows the results of all 10 rounds of this competition.

$$\begin{array}{r} \\ \text{Team 1} \\ \text{Team 2} \\ \text{Team 3} \\ \text{Team 4} \end{array} \begin{array}{c} \begin{array}{cccc} 1^{st} & 2^{nd} & 3^{rd} & 4^{th} \end{array} \\ \begin{bmatrix} 3 & 4 & 1 & 2 \\ 2 & 3 & 3 & 2 \\ 4 & 1 & 1 & 4 \\ 1 & 2 & 5 & 2 \end{bmatrix} \end{array}$$

Teams earn 4 points for each 1st place finish, 3 points for 2nd place finishes, 2 points for 3rd place finishes, and 1 point for 4th place finishes. Which teams tie for 2nd place?

A. 1 and 2

B. 2 and 3

C. 3 and 4

D. 4 and 1

8) A used bookstore sells paperback books for $1.00 each, hardback books for $3.00 each, and CDs for $4.00 each. On Saturday, they sold 37 paperbacks, 52 hardbacks, and 42 CDs. What matrix operation would compute the store's total income for that day?

A.

$$\begin{bmatrix} \$1.00 & \$3.00 & \$4.00 \end{bmatrix} \begin{bmatrix} 37 \\ 52 \\ 42 \end{bmatrix}$$

C.

$$\begin{bmatrix} 37 \\ 52 \\ 42 \end{bmatrix} \begin{bmatrix} \$1.00 & \$3.00 & \$4.00 \end{bmatrix}$$

B.

$$\begin{bmatrix} \$1.00 & \$3.00 & \$4.00 \end{bmatrix} [37 \quad 52$$

D. $$\begin{bmatrix} 37 \\ 52 \\ 42 \end{bmatrix} \begin{bmatrix} \$1.00 \\ \$3.00 \\ \$4.00 \end{bmatrix}$$

9) Carlos has investments in Funds A, B, and C. Each fund invests money in both stocks and bonds. The matrices show the dollar amounts invested in each fund and the annual yields. Use this information to determine how many dollars Fund B will earn in one year.

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \text{Stocks} \quad\quad \text{Bonds} \\ \begin{bmatrix} \$10{,}000 & \$10{,}000 \\ \$15{,}000 & \$\ 5{,}000 \\ \$\ 5{,}000 & \$25{,}000 \end{bmatrix} \end{array} \qquad \begin{array}{c} \text{Annual Yield} \\ \begin{array}{c} \text{Stocks} \\ \text{Bonds} \end{array} \begin{bmatrix} .06 \\ .04 \end{bmatrix} \end{array}$$

A. $ 1,100

B. $ 1,500

C. $ 2,000

D. $20,000

10) Matrix *A* represents the amount of fruit, in pounds, Juanita purchased on 3 different trips to a store. Matrix *B* gives the price per pound of each type of fruit.

$$A = \begin{bmatrix} \text{oranges} & \text{pears} & \text{grapes} \\ 6 & 7 & 2 \\ 12 & 5 & 1 \\ 18 & 0 & 3 \end{bmatrix}$$

$$B = \begin{matrix} \text{oranges} \\ \text{pears} \\ \text{grapes} \end{matrix} \begin{bmatrix} \$0.50 \\ \$2.00 \\ \$2.25 \end{bmatrix}$$

What matrix gives the amount Juanita spent on each trip?

A.

$$\begin{bmatrix} \$18.00 \\ \$24.00 \\ \$13.50 \end{bmatrix}$$

B.

$$\begin{bmatrix} \$21.50 \\ \$18.25 \\ \$15.75 \end{bmatrix}$$

C.

$$\begin{bmatrix} \$71.25 \\ \$85.50 \\ \$99.75 \end{bmatrix}$$

D.

$$\begin{bmatrix} \$3.00 & \$3.50 & \$1.00 \\ \$24.00 & \$10.00 & \$2.00 \\ \$40.50 & \$0.00 & \$6.75 \end{bmatrix}$$

11) Ashok, Kiri, and Justin designed 3 computer games, *x*, *y*, and *z*. They have sold some games and want to expand their business by advertising on the Internet. This matrix gives their sales for the first month.

$$\begin{matrix} x \\ y \\ z \end{matrix}\begin{bmatrix} 13 \\ 9 \\ 22 \end{bmatrix}$$

Their goal is to double the number of games sold each month for the next 4 months. What matrix represents their goal?

A.
$$\begin{bmatrix} 26 \\ 18 \\ 44 \end{bmatrix}$$

B.
$$\begin{bmatrix} 104 \\ 72 \\ 176 \end{bmatrix}$$

C.
$$\begin{bmatrix} 208 \\ 144 \\ 352 \end{bmatrix}$$

D.
$$\begin{bmatrix} 52 \\ 36 \\ 88 \end{bmatrix}$$

12) Evaluate $\begin{vmatrix} 4 & 9 \\ 3 & -6 \end{vmatrix}$.

A. −51
B. −3
C. 3
D. 54

13) Evaluate $\begin{vmatrix} 2 & 1 & 4 \\ 0 & 0 & 5 \\ 3 & -3 & 2 \end{vmatrix}$.

A. −15
B. 14
C. 15
D. 45

14) Find the inverse of $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$.

A. $\begin{bmatrix} 1 & -2 \\ -4 & 3 \end{bmatrix}$

C. $\begin{bmatrix} -5 & 10 \\ 20 & -15 \end{bmatrix}$

B. $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

D. $\begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$

15) For Matrix B to be the inverse of Matrix A, what must be the value of x in Matrix B ?

$$A = \begin{bmatrix} \frac{1}{5} & \frac{3}{10} \\ \frac{2}{5} & \frac{1}{10} \end{bmatrix} \qquad B = \begin{bmatrix} x & 3 \\ 4 & -2 \end{bmatrix}$$

A. −8
B. −6
C. −3
D. −1

16) The matrix $\begin{bmatrix} (x-2) & (x-2)^2 \\ (x+3) & (x+3)^3 \end{bmatrix}$ will NOT have an inverse for which nonnegative value of x ?

A. 0
B. 1
C. 2
D. 3

17) What is the inverse of this matrix?

$$\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

A. $\begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ -1 & -\frac{1}{2} \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$

B. $\begin{bmatrix} -\frac{1}{3} & -\frac{1}{4} \\ 1 & \frac{1}{2} \end{bmatrix}$

D. $\begin{bmatrix} -\frac{3}{2} & -2 \\ \frac{1}{2} & 1 \end{bmatrix}$

18) This table shows data from a geometry investigation.

x	4	5	6	7
y	2	5	9	14

If *y* is a quadratic function of *x*, which matrix equation could be used to find the equation of *y* ?

A. $\begin{bmatrix} 16 & 4 & 1 \\ 25 & 5 & 1 \\ 49 & 7 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 14 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 4 & 1 \\ 5 & 5 & 1 \\ 7 & 7 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$

B. $\begin{bmatrix} 16 & 4 & 1 \\ 25 & 5 & 1 \\ 49 & 7 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$

D. $\begin{bmatrix} 16 & 4 & 0 \\ 25 & 5 & 0 \\ 49 & 7 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 14 \end{bmatrix}$

19) Three friends went shopping at their favorite store. All the pants, shirts, and sweaters were on sale, and articles of each type cost the same. The table shows the friends' purchases.

Clothing Purchases and Costs				
	# of pants	# of shirts	# of sweaters	Total cost
Alicia	5	5	2	$377
Bette	5	6	0	$322
Cara	3	7	3	$408

Using the information in the table, determine the cost of each sweater.

A. $20.90

B. $32.00

C. $41.00

D. $45.45

20) If D, E, F, and X are 2 x 2 matrices, with

$$D = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 3 & -1 \\ 0 & 6 \end{bmatrix}$$

$$F = \begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}$$

and $DX + E = F$, what is X ?

A. $\begin{bmatrix} 1 & 10 \\ -15 & -24 \end{bmatrix}$

B. $\begin{bmatrix} -\frac{5}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{11}{4} \end{bmatrix}$

C. $\begin{bmatrix} -\frac{8}{13} & \frac{4}{13} \\ -\frac{3}{13} & -\frac{18}{13} \end{bmatrix}$

D. $\begin{bmatrix} -\frac{9}{52} & -\frac{11}{156} \\ \frac{2}{13} & \frac{1}{39} \end{bmatrix}$

21) Given:

$\begin{cases} Ax + By = p \\ Cx + Dy = q \end{cases}$ is a system of linear equations

such that $\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = 0$

Which statement *must* be true?

A. The system has no solution.

B. The system has infinitely many solutions.

C. The system has one unique solution, based on the values of p and q.

D. The system has either no solution or many solutions, depending on the values of p and q.

22)

What is the inverse of $\begin{bmatrix} 2 & 1 & -1 \\ -2 & 2 & 2 \\ -1 & 2 & 1 \end{bmatrix}$?

A. $\begin{bmatrix} 1 & 1.5 & 2 \\ 0 & -0.5 & 1 \\ 1 & 2.5 & -3 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1.5 & -2 \\ 0 & -0.5 & 1 \\ 1 & 2.5 & -3 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -1.5 & 2 \\ 0 & 1.5 & -1 \\ -1 & 2.5 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1.5 & -2 \\ 1 & -0.5 & 1 \\ 0 & 2.5 & -3 \end{bmatrix}$

23) An unusual aspect of 2×2 matrices is that they can be both added and also multiplied, and the result in all cases is a 2×2 matrix. Tabitha asserts that the product $\left(\begin{bmatrix} 4 & -1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -2 & 8 \end{bmatrix}\right)\begin{bmatrix} 7 & 0 \\ 1 & -3 \end{bmatrix}$ equals the sum $\begin{bmatrix} 4 & -1 \\ 3 & 0 \end{bmatrix} + \left(\begin{bmatrix} 2 & 3 \\ -2 & 8 \end{bmatrix} \bullet \begin{bmatrix} 7 & 0 \\ 1 & -3 \end{bmatrix}\right)$. Show that she is mistaken.

24)

Among the matrices $\begin{bmatrix} 4 & -1 \\ 3 & 1 \end{bmatrix}$, $\begin{bmatrix} 4 & 0 & 1 & 2 & -3 \\ 1 & -2 & 7 & 0 & -9 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 7 & -1 \\ -14 & 2 \end{bmatrix}$, one has an inverse.

Which matrix, and what is the inverse?

25) Cara's teacher lists these 2 matrices as class examples. The teacher says that one of these matrices has an inverse and one does not.

$$J = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix} \qquad K = \begin{bmatrix} 6 & -3 \\ 4 & 2 \end{bmatrix}$$

A. What is the determinant of each matrix? Show your algebraic work.

B. Cara says that J has an inverse but K does not. Do you agree with Cara? Why or why not?

C. Find the inverse of the matrix that has an inverse. Show your algebraic work. Explain how you found the inverse as if you were explaining the process to a classmate who missed class when you learned this concept.

26) Consider the given matrices:

$$H = \begin{bmatrix} -2 & 0 & 4 \\ 1 & 3 & 0 \end{bmatrix} \quad J = \begin{bmatrix} 4 & 3 & -1 \\ 2 & 4 & 5 \end{bmatrix} \quad K = \begin{bmatrix} 0 & -2 \\ 1 & 5 \end{bmatrix}$$

A. What matrix, G, would be added to H to obtain J ? Show your work algebraically.

B. What is $H \bullet K$, if it can be computed? Show your work algebraically, and explain the approach you used to find your answer. If the expression cannot be computed, explain why not.

C. What is $K \bullet J$, if it can be computed? Show your work algebraically, and explain the approach you used to find your answer. If the expression cannot be computed, explain why not.

Please use the space below to write your response(s) to the writing assignment provided by your teacher. If there are multiple tasks to the question, please clearly label the number or letter of each task in the column to the left of your answers. If you need additional pages for your response, your teacher can provide them.

Please write the name of the writing assignment here: ____

Task

QualityCore™

Reference Sheet for the QualityCore™
Algebra II End-of-Course Assessment

Equations of a Line

Standard Form	$Ax + By = C$	A, B, and C are constants with A and B not both equal to zero.
Slope-Intercept Form	$y = mx + b$	(x_1, y_1) is a point.
Point-Slope Form	$y - y_1 = m(x - x_1)$	m = slope b = y-intercept

Quadratics

Standard Form of a Quadratic Equation	$ax^2 + bx + c = 0$	a, b, and c are constants, where $a \neq 0$.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Conic Sections

Circle	$(x - h)^2 + (y - k)^2 = r^2$	center (h,k) r = radius	
Parabola	$y = a(x - h)^2 + k$	axis of symmetry $x = h$ directrix $y = k - \frac{1}{4a}$	vertex (h,k) focus $\left(h, k + \frac{1}{4a}\right)$
Parabola	$x = a(y - k)^2 + h$	axis of symmetry $y = k$ directrix $x = h - \frac{1}{4a}$	vertex (h,k) focus $(h + \frac{1}{4a}, k)$
Ellipse	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$, center (h,k)	
Ellipse	$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 - b^2$, center (h,k)	
Hyperbola	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$, center (h,k)	
Hyperbola	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 + b^2$, center (h,k)	

Lines and Points

Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	(x_1, y_1) and (x_2, y_2) are 2 points. m = slope
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	M = midpoint d = distance
Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	

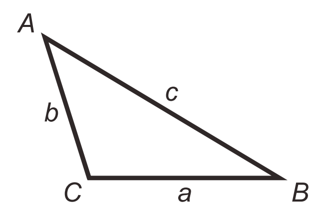
continued

Miscellaneous

Distance, Rate, Time	$D = rt$	D = distance
Simple Interest	$I = prt$	r = rate
Compound Interest	$A = p\left(1 + \frac{r}{n}\right)^{nt}$	t = time
		I = interest
		p = principal
		A = amount of money after t years
		n = number of times interest is compounded annually
Pythagorean Theorem	$a^2 + b^2 = c^2$	a and b = legs of right triangle
		c = hypotenuse

Laws of Sines and Cosines

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$	



Sequences, Series, and Counting

Arithmetic Sequence	$a_n = a_1 + (n - 1)d$	$a_n = n^{th}$ term
Arithmetic Series	$s_n = \frac{n}{2}(a_1 + a_n)$	n = number of the term
Geometric Sequence	$a_n = a_1(r^{n-1})$	d = common difference
Geometric Series	$s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$	s_n = sum of the first n terms
Combinations	${}_kC_m = C(k,m) = \frac{k!}{(k-m)!\,m!}$	r = common ratio
Permutations	${}_kP_m = P(k,m) = \frac{k!}{(k-m)!}$	k = number of objects in the set
		m = number of objects selected

Circumference, Area, and Volume

Triangle	$A = \frac{1}{2}bh$	A = area
Parallelogram	$A = bh$	b = base
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	h = height
Circle	$A = \pi r^2$ $C = \pi d$	r = radius
General Prism	$V = Bh$	C = circumference
Right Circular Cylinder	$V = \pi r^2 h$	d = diameter
Pyramid	$V = \frac{1}{3}Bh$	V = volume
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	B = area of base
Sphere	$V = \frac{4}{3}\pi r^3$	$\pi \approx 3.14$

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Answer Key

1) B
2) D
3) A
4) C
5) D
6) A
7) B
8) A
9) A
10) B
11) C
12) A
13) D
14) D
15) D
16) C
17) C
18) A
19) C
20) C
21) D
22) C

Answer:

23) No. The product is $\begin{bmatrix} 44 & -6 \\ 15 & -24 \end{bmatrix}$ which is not equal to the sum: $\begin{bmatrix} 21 & -10 \\ -3 & -24 \end{bmatrix}$.

Answer:

24) For a matrix to have an inverse, it must meet two conditions. It must be a square matrix, and its determinant must not be 0.

The determinant of $\begin{bmatrix} 4 & -1 \\ 3 & 1 \end{bmatrix}$ = (4)(1) − (−1)(3) = 7, and its inverse is $\begin{bmatrix} \frac{1}{7} & \frac{1}{7} \\ -\frac{3}{7} & \frac{4}{7} \end{bmatrix}$.

Scoring Criteria:

25)

A 4-point response may include, but is not limited to, the following points:

A. **Correct determinant for *J*:** 0

Correct determinant for *K*: 24

Appropriate work leading to the answer:

$\det J = 3(4) - 2(6) = 12 - 12$

$\det K = 6(2) - (-3)(4) = 12 + 12$

B. **Agreement with Cara:** No

Explanation of decision: A matrix has an inverse only when the determinant of that matrix is not equal to zero. Since the determinant of *J* is zero, *J* does not have an inverse. Since *K* has a determinant of 24 (which is not equal to zero), then *K* does have an inverse.

C. **Correct inverse:** $K^{-1} = \begin{bmatrix} \frac{1}{12} & \frac{1}{8} \\ -\frac{1}{6} & \frac{1}{4} \end{bmatrix}$

Appropriate work leading to the answer:

$$\begin{bmatrix} 6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6a - 3c & 6b - 3d \\ 4a + 2c & 4b + 2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} 6a - 3c = 1 \\ 4a + 2c = 0 \end{cases} \text{ and } \begin{cases} 6b - 3d = 0 \\ 4b + 2d = 1 \end{cases}$$

$$\begin{cases} 6a - 3c = 1 \\ 2a + c = 0 \end{cases} \text{ and } \begin{cases} 2b - d = 0 \\ 4b + 2d = 1 \end{cases}$$

$$\begin{array}{rcrcl|rcrcl} 6a & - & 3c & = & 1 & 6a & - & 3c & = & 1 \\ 2a & + & c & = & 0 \xrightarrow{\times 3} & 6a & + & 3c & = & 0 \\ \hline & & & & & 12a & & & = & 1 \end{array}$$

and

$$\begin{array}{rcrcl|rcrcl} 2b & - & d & = & 0 \xrightarrow{\times 2} & 4b & - & 2d & = & 0 \\ 4b & + & 2d & = & 1 & 4b & + & 2d & = & 1 \\ \hline & & & & & 8b & & & = & 1 \end{array}$$

$a = \frac{1}{12}$ and $b = \frac{1}{8}$

$2\left(\frac{1}{12}\right) + c = 0$ and $2\left(\frac{1}{8}\right) - d = 0$

$\frac{1}{6} + c = 0$ and $\frac{1}{4} - d = 0$

$c = -\frac{1}{6}$ and $d = \frac{1}{4}$

OR

$$\frac{1}{24}\begin{bmatrix} 2 & 3 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} \frac{2}{24} & \frac{3}{24} \\ \frac{-4}{24} & \frac{6}{24} \end{bmatrix} = \begin{bmatrix} \frac{1}{12} & \frac{1}{8} \\ -\frac{1}{6} & \frac{1}{4} \end{bmatrix}$$

Explanation of how the inverse was found: I multiplied K by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and set the product equal to the identity matrix. Then, I formed 2 systems of equations by setting corresponding elements of the matrices equal. I formed the systems by keeping the equations with the same variables in one system. I divided through by 2 in the equations that are equal to 0 to simplify the calculations. Then, I used elimination to solve the systems of equations. I had to multiply one equation by 3 to cancel $-3c$ and $3c$ so that I could solve for a. I had to multiply another equation by 2 to cancel $-2d$ and $2d$ so that I could solve for b. I substituted a and b back into the simplified equations that were equal to 0 and solved for c and d.

OR

I used $\frac{1}{\det K}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, substituted my values from matrix K, and simplified.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

Scoring Criteria:

26)

A 4-point response may include, but is not limited to, the following points:

A. **Correct matrix:** $G=\begin{bmatrix}6 & 3 & -5\\ 1 & 1 & 5\end{bmatrix}$

Appropriate work leading to the answer:

$$\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}+\begin{bmatrix}-2 & 0 & 4\\ 1 & 3 & 0\end{bmatrix}=\begin{bmatrix}4 & 3 & -1\\ 2 & 4 & 5\end{bmatrix}$$

$$\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}=\begin{bmatrix}4 & 3 & -1\\ 2 & 4 & 5\end{bmatrix}-\begin{bmatrix}-2 & 0 & 4\\ 1 & 3 & 0\end{bmatrix}$$

$$\begin{bmatrix}a & b & c\\ d & e & f\end{bmatrix}=\begin{bmatrix}4-(-2) & 3-0 & -1-4\\ 2 & -1 & 4-3 & 5-0\end{bmatrix}$$

B. **Correct product for *H* • *K*:** The matrix product cannot be computed.

Explanation of why the expression cannot be computed: Matrix *H* is a 2×3 matrix and matrix *K* is a 2×2 matrix. The expression *H* • *K* cannot be computed because the inner dimensions of the product are not equal: $(2\times3)(2\times2)$; $3\neq2$. If I try to multiply *H* by *K*, I will run out of elements in the columns of *K* before I use all the elements in the rows of *H*.

C. **Correct product of *K* • *J*:** $\begin{bmatrix}-4 & -8 & -10\\ 14 & 23 & 24\end{bmatrix}$

Appropriate work leading to the answer:

$$\begin{bmatrix}0 & -2\\ 1 & 5\end{bmatrix}\begin{bmatrix}4 & 3 & -1\\ 2 & 4 & 5\end{bmatrix}=\begin{bmatrix}0(4)+(-2)(2) & 0(3)+(-2)(4) & 0(-1)+(-2)(5)\\ 1(4)+5(2) & 1(3)+5(4) & 1(-1)+5(5)\end{bmatrix}$$

Explanation of the approach used to find the answer: I multiplied the matrices together by multiplying across the rows of the first matrix and down the columns of the second matrix.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**
- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**
- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**
- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**
- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.