Name: Date:
Teacher: Class/Period:

1) What is the nature of the zeros of the polynomial $f(x) = 2x^3 - x^2 - 18x + 9$?

A. 3 real rational

B. 3 real; 1 rational and 2 irrational

C. 1 real rational, 2 nonreal complex

D. 1 real irrational, 2 nonreal complex

2) A fourth degree polynomial, $P(x)$, with real coefficients has 4 distinct zeros. Two of them are -5 and i. What can be concluded about the other zeros?

A. The other zeros must be 5 and $-i$.

B. One of the other zeros must be $-5 \pm i$.

C. One of the other zeros must be $5 \pm i$.

D. The other zeros must be $-i$ and a real number.

3) How many real zeros does $h(t)$ have?

$$h(t) = 4t^3 - 2t^2 + t - 10$$

A. 3

B. 2

C. 1

D. 0

4) What are the rational zeros for $x^3 - 3x^2 - 4x + 12$?

A. $-2, 2, 3$

B. $-2, 2, -3$

C. 1, 2, 3, 4, 6, 12

D. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

5) How many times does $f(x) = 4x^3 - 3x$ cross the x-axis?

A. 2

B. 3

C. 4

D. 5

6) How many real roots does $f(x) = x^3 + 2x^2 + x$ have?

A. 0

B. 1

C. 2

D. 3

7) What is the number of real roots of this equation?

$$(x^4 + 1)(x - \sqrt{3}) = 0$$

A. 1

B. 2

C. 4

D. 5

8) How many rational zeros does this polynomial function have?

$$f(x) = (x^4 - 16)(3x^2 - 21)(4x^2 + 1)$$

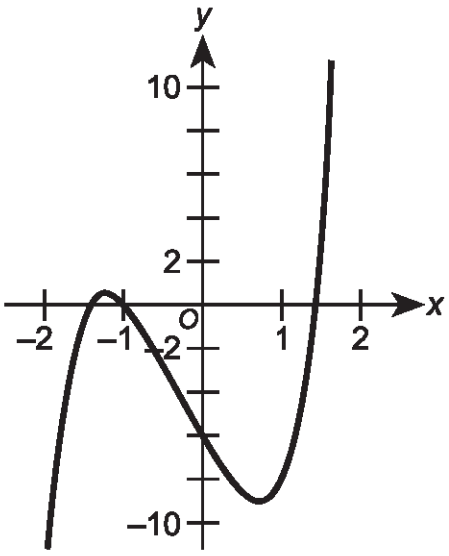
A. 8

B. 6

C. 4

D. 2

9) The graph shows a fifth degree polynomial function $y = p(x)$.



What is the nature of the roots of $y = p(x)$?

A. 3 real and 0 imaginary

B. 3 real and 2 imaginary

C. 0 real and 5 imaginary

D. 5 real and 0 imaginary

10) Use Descartes' Rule of Signs to determine the possible number of negative real roots of $f(x) = x^3 + 4x^2 - 2x - 1$.

A. 6

B. 4

C. 3 or 1

D. 2 or 0

11) How many rational zeros does this function have?

$$f(x) = (x^2 + 4)(x - 4)(x^2 - 2x - 5)$$

A. 1

B. 2

C. 3

D. 5

12) Which of the following lists best describes the five complex zeros of the function $2x^5 + 3x^4 + 11x^3 + 24x^2 - 63x - 27$?

A. Three rational, two nonreal

B. Two rational, one irrational, two nonreal

C. One rational, two irrational, two nonreal

D. One rational triple root, two irrational

13) A polynomial function, $P(x)$, has integer coefficients. The leading coefficient, R, has 4 positive factors, and the constant term, S, has 2 positive factors. R and S have one common factor other than 1. According to the Rational Roots Theorem, what is the maximum number of possible rational zeros of $P(x)$?

A. 3

B. 8

C. 12

D. 32

14) Find all zeros of $f(x) = x^4 - 1$.

A. $x = -1, 1$

B. $x = 1, i$

C. $x = 1, i, -i$

D. $x = 1, -1, i, -i$

15) Maury is standing on top of a building. He throws a ball up from the top of the building. The height of the ball is modeled by $h(x) = -16t^2 + 12t + 40$. How many seconds after release will the ball hit the ground?

A. -2

B. $-\frac{5}{4}$

C. $\frac{5}{4}$

D. 2

16) Given the polynomial function $f(x) = ax^3 - acx^2 - bx^2 + bcx$, where a, b, and c are nonzero numbers, find all zeros.

A. $x = 0, \frac{b}{a}, c$

B. $x = 0, \frac{a}{c}, b$

C. $x = 0, \frac{a}{b}, c$

D. $x = 0, \frac{b}{c}, a$

17) What are the possible rational solutions of this equation?

$$\frac{1}{2}x^5 + 3x^4 - 4x^3 + 6x = 2$$

A. $\pm 1, \pm 2$

B. $\pm 1, \pm 2, \pm 4$

C. $\pm 1, \pm 2, \pm 3, \pm 6$

D. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

18) What are the rational roots of the function $f(x) = x^4 + 2x^2 - 3$?

A. $-1, 1$

B. $-1, 1, 3, -3$

C. $-1, 1, \sqrt{3}, -\sqrt{3}$

D. $3, -3$

19) Which cubic polynomial has 3 and $3 - i$ as zeros?

A. $x^3 - 3x^2 - 9x + 27$

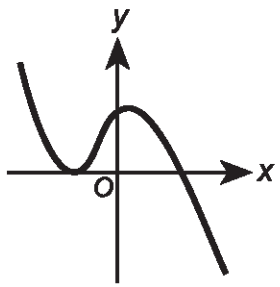
B. $x^3 + 3x^2 - 10x - 30$

C. $x^3 - 9x^2 + 28x - 30$

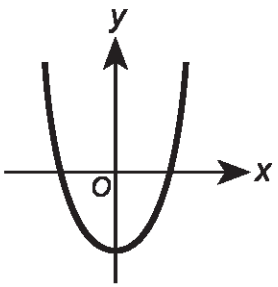
D. $x^3 + 9x^2 + 28x + 30$

20) Which graph could represent a cubic function with 2 distinct real zeros?

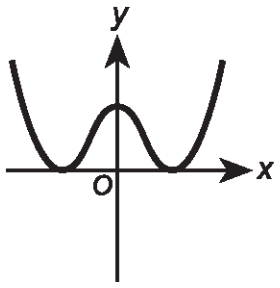
A.



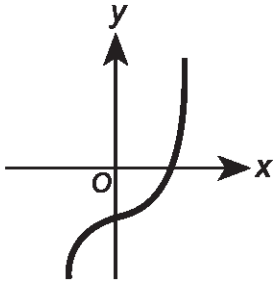
C.



B.



D.



21) Julio throws an inflated ball up in the air. The function $h(t) = -\frac{1}{20}t^2 + \frac{1}{10}t + 4$ models the ball's height in terms of time t, in seconds. After how many seconds will the ball hit the ground?

A. 0

B. 4

C. 8

D. 10

22) Given that $x = c$ is a root of a polynomial function, $f(x)$, and that $h(x) = 3f(x)$, which is always a factor of $h(x)$?

A. $x - c$

B. $x - 3c$

C. $x - \frac{1}{c}$

D. $x - \frac{1}{3}c$

23) For what values of x does the graph of $f(x) = 3x^2 - 5\frac{1}{2}x - 5$ intersect the x-axis?

A. −11 and 5

B. 11 and −5

C. $-2\frac{1}{2}$ and $\frac{2}{3}$

D. $2\frac{1}{2}$ and $-\frac{2}{3}$

24) The zeros of the polynomial function $f(x) = x^3 + bx^2 + cx + d$ are 2, 1, and −1. Which equation could be used to represent $f(x)$?

A. $f(x) = x^3 + 2x^2 + x - 2$

B. $f(x) = x^3 - 2x^2 - x + 2$

C. $f(x) = x^3 + 2x^2 - x - 2$

D. $f(x) = x^3 - 2x^2 + x + 2$

25) A certain polynomial has these factors and no others: $(3x - 2)$, $(x^2 - 1)$, and $(x + 4)$. What are the zeros of this polynomial?

A. 4, −2, −1

B. 4, 1, 2

C. $-4, \frac{2}{3}, 1$

D. $-4, -1, \frac{2}{3}, 1$

26) A certain seventh degree polynomial function can be factored as:

$$(x + 2)^2(x^2 + 1)(x - 5)^3$$

How many x-intercepts does this function have?

A. 2

B. 3

C. 5

D. 7

27) How many points of intersection with the x-axis does this function have?

$$f(x) = (x^2 + 2)(x^2 - 3x + 4)(x^3 - 8)$$

A. 1

B. 3

C. 5

D. 7

28) This polynomial equation

$$3x^3 - 8x^2 - 185x + 126 = 0$$

has 3 real solutions. Two of the solutions are integers but the other is not. The noninteger solution is between which 2 consecutive integers?

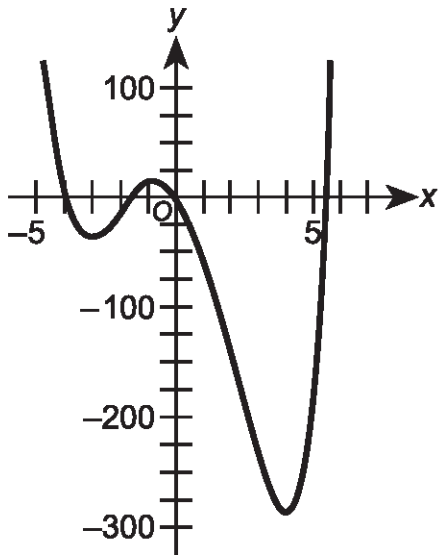
A. −1 and 0

B. 0 and 1

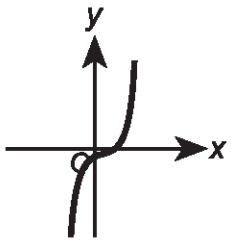
C. 5 and 6

D. 8 and 9

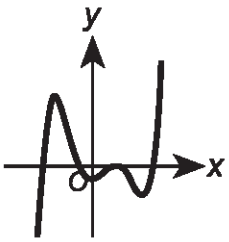
29) Consider the graph of the function $f(x) = x^4 - 25x^2 - 36x$, which has one x-intercept at (−4,0). Find all the other zeros of the function algebraically. Show your work, and explain the approach you used to find your answer.



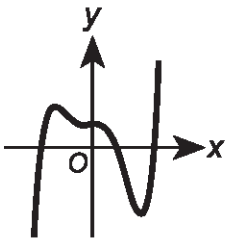
30) Ms. Phillips explained to her class that polynomial functions of degree 5 with real coefficients always have 5 roots. She showed her students the following 3 graphs of polynomial functions of degree 5.



Graph 1



Graph 2



Graph 3

A. State how many distinct real roots there are for each graphed polynomial. Use evidence from the graphs to explain your answer.

B. Each of the graphed polynomials has a different number of real roots. State how many real and non-real roots each graphed polynomial has. Use evidence from the graphs to explain your answer.

31) Given the function $f(x) = -x^4 - 3x^3 + 101x^2 + 543x + 940$, use a graphing calculator to do the following:

A. Find the zeros of the function. Round any zeros you find to the nearest tenth. Explain how you found your answer step-by-step, as if you were explaining to a student who does not know how to use a graphing calculator.

B. Identify any local minima or maxima of the graph of the function as ordered pairs. Indicate which are local minima and which are local maxima. Round any minima or maxima to the nearest tenth. Explain how you found your answer step-by-step.

C. Find the range of the function. Round the numbers in your answer to the nearest tenth. Explain how you found your answer step-by-step.

D. Sketch the graph of the function. List a viewing window that shows all the information you found in the other parts of the problem. Explain why you chose your viewing window.

Please use the space below to write your response(s) to the writing assignment provided by your teacher. If there are multiple tasks to the question, please clearly label the number or letter of each task in the column to the left of your answers. If you need additional pages for your response, your teacher can provide them.

Please write the name of the writing assignment here: _____

Task

QualityCore™

Reference Sheet for the QualityCore™ **Algebra II End-of-Course Assessment**

Equations of a Line

Standard Form	$Ax + By = C$	A, B, and C are constants with A and B not both equal to zero.
Slope-Intercept Form	$y = mx + b$	(x_1,y_1) is a point. m = slope b = y-intercept
Point-Slope Form	$y - y_1 = m(x - x_1)$	

Quadratics

Standard Form of a Quadratic Equation	$ax^2 + bx + c = 0$	a, b, and c are constants, where $a \neq 0$.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Conic Sections

Circle	$(x - h)^2 + (y - k)^2 = r^2$	center (h,k) r = radius	
Parabola	$y = a(x - h)^2 + k$	axis of symmetry $x = h$ directrix $y = k - \frac{1}{4a}$	vertex (h,k) focus $\left(h, k + \frac{1}{4a}\right)$
Parabola	$x = a(y - k)^2 + h$	axis of symmetry $y = k$ directrix $x = h - \frac{1}{4a}$	vertex (h,k) focus $\left(h + \frac{1}{4a}, k\right)$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$, center (h,k)	
Ellipse	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 - b^2$, center (h,k)	
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$, center (h,k)	
Hyperbola	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 + b^2$, center (h,k)	

Lines and Points

Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	(x_1,y_1) and (x_2,y_2) are 2 points. m = slope M = midpoint d = distance
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	
Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	

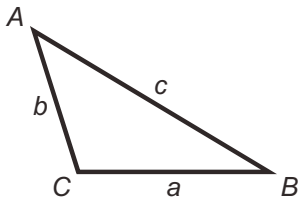
continued

Miscellaneous

Distance, Rate, Time	$D = rt$	D = distance r = rate t = time I = interest p = principal A = amount of money after t years n = number of times interest is compounded annually
Simple Interest	$I = prt$	
Compound Interest	$A = p\left(1 + \frac{r}{n}\right)^{nt}$	
Pythagorean Theorem	$a^2 + b^2 = c^2$	a and b = legs of right triangle c = hypotenuse

Laws of Sines and Cosines

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$



Sequences, Series, and Counting

Arithmetic Sequence	$a_n = a_1 + (n - 1)d$	$a_n = n^{\text{th}}$ term n = number of the term d = common difference s_n = sum of the first n terms r = common ratio k = number of objects in the set m = number of objects selected
Arithmetic Series	$s_n = \frac{n}{2}(a_1 + a_n)$	
Geometric Sequence	$a_n = a_1(r^{n-1})$	
Geometric Series	$s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$	
Combinations	${}_kC_m = C(k,m) = \frac{k!}{(k-m)!\,m!}$	
Permutations	${}_kP_m = P(k,m) = \frac{k!}{(k-m)!}$	

Circumference, Area, and Volume

Triangle	$A = \frac{1}{2}bh$	A = area b = base h = height r = radius C = circumference d = diameter V = volume B = area of base $\pi \approx 3.14$
Parallelogram	$A = bh$	
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	
Circle	$A = \pi r^2$ $C = \pi d$	
General Prism	$V = Bh$	
Right Circular Cylinder	$V = \pi r^2 h$	
Pyramid	$V = \frac{1}{3}Bh$	
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	
Sphere	$V = \frac{4}{3}\pi r^3$	

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Answer Key

1) A
2) D
3) C
4) A
5) B
6) C
7) A
8) D
9) B
10) D
11) A
12) C
13) C
14) D
15) D
16) A
17) B
18) A
19) C
20) A
21) D
22) A
23) D
24) B
25) D
26) A
27) A
28) B

Scoring Criteria:

29)

A 4-point response may include, but is not limited to, the following points:

Correct additional zeros: $0,\ 2\pm\sqrt{13}$

Appropriate work needed to find the answer:

$f(x)=x^4-25x^2-36x=x(x^3-25x-36)$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -25 & -36 \\ & & -4 & 16 & 36 \\ \hline & 1 & -4 & -9 & 0 \end{array}$$

$x^2-4x-9=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-9)}}{2(1)}=\frac{4\pm\sqrt{16+36}}{2}=\frac{4\pm\sqrt{52}}{2}=\frac{4\pm2\sqrt{13}}{2}=2\pm\sqrt{13}$$

Note: An examinee could also use polynomial long division instead of synthetic division to find the answer.

Explanation of the approach used to find the answer: First, I factored out an x. That gave me $x=0$ as a zero of the function. Since $(-4,0)$ is an x-intercept of the graph of the function, $x=-4$ is a zero of the function. Since $x=-4$ is a zero of the function, I used synthetic division to find $\frac{x^3-25x-36}{x+4}$. I put in 0 for the coefficient of the x^2 term in the synthetic division process. Performing synthetic division told me that $\frac{x^3-25x-36}{x+4}=x^2-4x-9$. Then, I used the quadratic formula to find the remaining zeros.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

Scoring Criteria:

30) A 4-point response may include, but is not limited to, the following points:

A. **Correct number of distinct real roots for each graph:**
Graph 1: 1 distinct real root
Graph 2: 4 distinct real roots
Graph 3: 3 distinct real roots

Explanation of how the information shown in the graph proves this:
I examined each graph and looked at how many times it intersected with the *x*-axis. The number of times that each graph touches the *x*-axis gives the number of distinct real roots. Graph 1 intersected the *x*-axis 1 time. Graph 2 intersected the *x*-axis 4 times. Graph 3 intersected the *x*-axis 3 times.

B. **Correct numbers of real and non-real roots:**
Graph 1: 1 real root and 4 non-real roots
Graph 2: 5 real roots
Graph 3: 3 real roots and 2 non-real roots

Explanation of how the information shown in the graph proves this:
Graph 1 has 1 distinct real root. There can only be an even number of non-real roots, so Graph 1 must have 4 non-real roots.

Graph 2 has 4 distinct real roots. The graph intersects the *x*-axis 4 times, but one of the positive *x*-intercepts intersects but does not cross the *x*-axis, which means it is a double root. That means there are 5 real roots and 0 non-real roots.

Graph 3 crosses the *x*-axis 3 times and has no double roots, so there must be 3 real roots and 2 non-real roots.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

Scoring Criteria:

31)

A 4-point response may include, but is not limited to, the following points:

A. **Correct zeros:** $x \approx -8.8$ and $x \approx 11.1$

Explanation of how the answer was found: I used the zero function on my calculator. For the zeros of the graph, I put the cursor to the left of the zero for the lower bound and to the right of the zero for the upper bound and then let the calculator find the zero.

B. **Correct minimum:** $(-2.8, 215.7)$

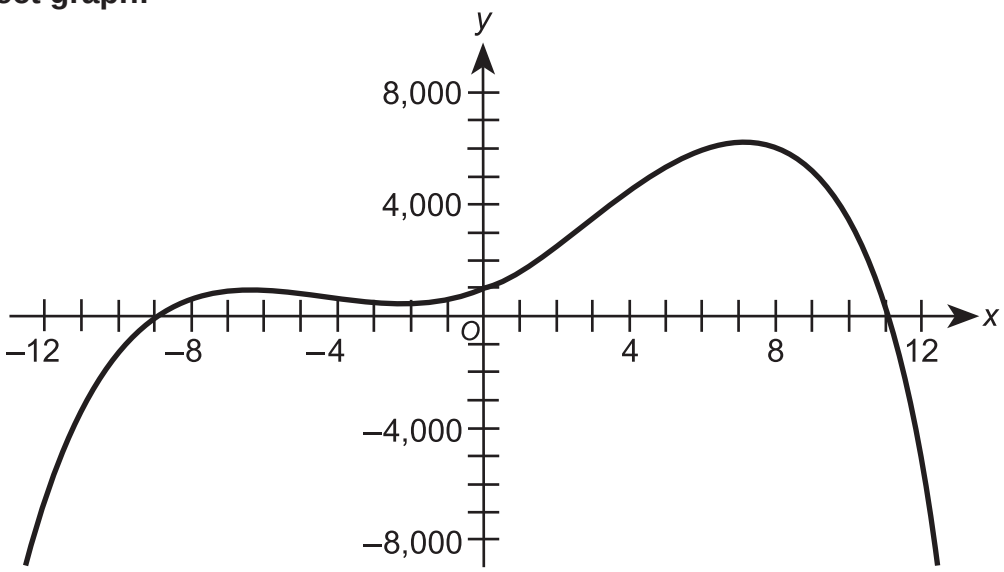
Correct maxima: $(-6.8, 723.3)$ and $(7.3, 6279.6)$

Explanation of how the answer was found: I used the minimum and maximum functions on my calculator. I moved the lower bound and upper bound such that they were on either side of the minimum or maxima. (I thought of a vertical line running through the points representing the lower and upper bounds and the minimum and maxima.)

C. **Correct range:** $(-\infty, 6279.6]$

Explanation of how the answer was found: I used the highest maximum as the highest value on the graph. Therefore, the range was all numbers less than or equal to that maximum value.

D. **Correct graph:**



Correct viewing window: $[-15, 15]$ by $[-8000, 8000]$

Explanation of why the viewing window was chosen: I chose a viewing window that contained both zeros (−8.8 and 11.1) for the *x*-coordinates and that contained the minimum and highest maximum (215.7 and 6279.6) for the *y*-coordinates. I decided I should go below the *x*-axis for the graph, rather than just going a little below the minimum so that the graph would be clearly identified in relation to the origin.

Note: Any viewing window that contains the zeros for the *x*-coordinates and the minimum and the highest maximum for the *y*-coordinates would be acceptable. The viewing window should go at least a little outside these values though, as cutting it off precisely at the given values would not show a good image of the whole graph.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.