Name: Date:
Teacher: Class/Period:

1) The equation $\frac{(x-4)^2}{c^2}+\frac{(y-6)^2}{d^2}=36$ describes an ellipse with the center at (4,6), a vertical major axis with a length of 6, and a horizontal minor axis with a length of 4. What are the values of c and d ?

A. $c=2$ and $d=3$

B. $c=3$ and $d=2$

C. $c=\frac{1}{2}$ and $d=\frac{1}{3}$

D. $c=\frac{1}{3}$ and $d=\frac{1}{2}$

2) What is the equation of a parabola that has its vertex at the origin and a directrix of $x=4$?

A. $y^2=-16x$

B. $y^2=-4x$

C. $x^2=-16y$

D. $x^2=-4y$

3) Which conic section is given by the equation $(x-h)^n=j(y-k)^m-12$ when $n=2$, $m=1$, and $j=-2$?

A. Circle

B. Ellipse

C. Hyperbola

D. Parabola

4) What are the x-intercepts of the ellipse with equation $9x^2+4y^2=36$?

A. 2, −2

B. 3, −3

C. 4, −4

D. 6, 0

5) This equation represents which conic section?

$$\frac{y^2}{74} = \frac{30 - x^2}{30}$$

A. A parabola that opens up

B. A parabola that opens down

C. An ellipse with a vertical major axis

D. An ellipse with a horizontal major axis

6) The equation $5x^2 + 4y^2 = 180$ represents what conic section?

A. Circle

B. Ellipse

C. Hyperbola

D. Parabola

7) What conic section does this equation represent, and what is its center?

$$\frac{(x+2)^2}{9} - \frac{(y-1)^2}{10} = 1$$

A. Hyperbola with center at $(2,-1)$

B. Hyperbola with center at $(-2,1)$

C. Ellipse with center at $(2,-1)$

D. Ellipse with center at $(-2,1)$

8) What is the standard form of the equation $25x^2 + y^2 + 100x - 2y + 76 = 0$?

A. $(x+2)^2 + \frac{(y-1)^2}{5} = 1$

B. $(x+2)^2 + \frac{(y+1)^2}{5} = 1$

C. $(x+2)^2 + \frac{(y-1)^2}{25} = 1$

D. $(x+2)^2 + \frac{(y+1)^2}{25} = 1$

9) What is the equation of the circle with center (3,−6) and radius $\frac{15}{8}$?

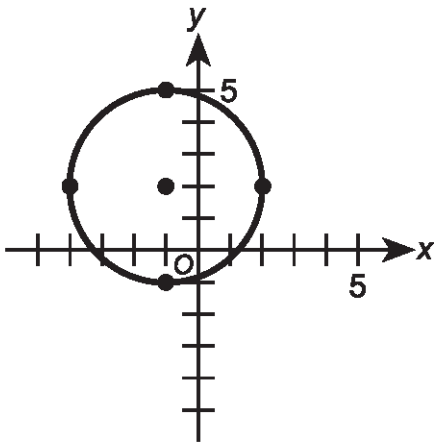
A. $(x-3)^2+(y+6)^2=\frac{225}{64}$

B. $(x-3)^2+(y+6)^2=\frac{15}{8}$

C. $(x+3)^2+(y-6)^2=\frac{225}{64}$

D. $(x+3)^2+(y-6)^2=\frac{15}{8}$

10) What is the equation of this circle?



A. $(x+1)^2+(y-2)^2=6$

B. $(x-1)^2+(y+2)^2=6$

C. $(x+1)^2+(y-2)^2=9$

D. $(x-1)^2+(y+2)^2=9$

11) A circle with equation $x^2+y^2=37$ has center (0,0). The circle is shifted 3 units to the right and 4 units down. What is the equation of the new circle?

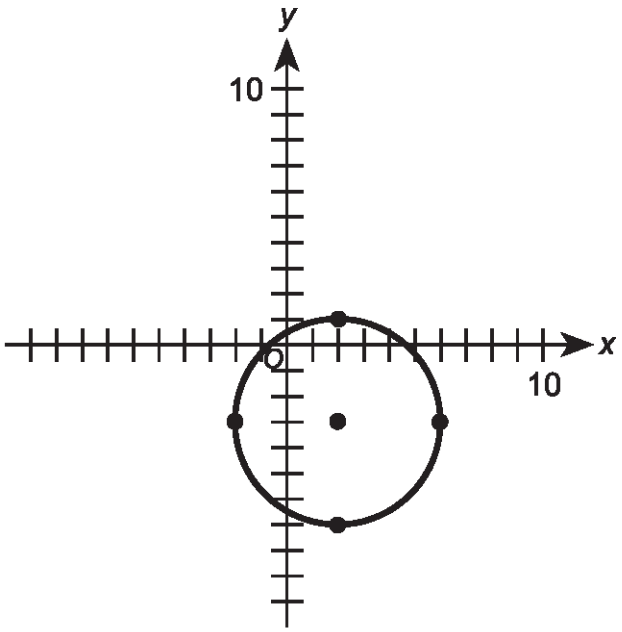
A. $(x+4)^2+(y-3)^2=37$

B. $(x-4)^2+(y+3)^2=37$

C. $(x+3)^2+(y-4)^2=37$

D. $(x-3)^2+(y+4)^2=37$

12) Find the equation of this circle.



A. $(x - 2)^2 + (y + 3)^2 = 4$

B. $(x - 2)^2 - (y + 3)^2 = 4$

C. $(x - 2)^2 + (y + 3)^2 = 16$

D. $(x - 2)^2 - (y + 3)^2 = 16$

13) Which phrase best describes the points of intersection between the graphs of $x^2 + y^2 + 2x - 16y + 16 = 0$ and $y^2 + 4x = 0$ in the xy-plane?

A. Two points of intersection, in Quadrant I

B. Two points of intersection, in Quadrant II

C. Four points of intersection, two in Quadrant II and two in Quadrant III

D. No points of intersection

14) The equation of a circle that is tangent to both the x-axis and the y-axis is given as $(x - a)^2 + (y - b)^2 = c$. What must always be true about the values of a, b, and c ?

A. $a = b$ and $a^2 + b^2 = c$

B. $a = b$ and $c = a^2$

C. $|a| = |b|$ and $a^2 + b^2 = c$

D. $|a| = |b|$ and $c = a^2$

15) This is the equation of a parabola:

$$y = -2x^2 + 4x + 2$$

Determine the x-value for the vertex and whether this value is a maximum or a minimum.

A. Maximum at $x = 1$

B. Maximum at $x = 2$

C. Minimum at $x = 1$

D. Minimum at $x = 2$

16) What is the vertex of the parabola with equation $(y + 2)^2 = 4(x + 3)$?

A. (2,3)

B. (3,2)

C. (−2,−3)

D. (−3,−2)

17) What is the standard form of this equation of a circle?

$$3x^2 + 3y^2 - 6x + 18y + 18 = 0$$

A. $(x - 1)^2 + (y + 3)^2 = 4$

B. $(x + 1)^2 + (y - 3)^2 = 4$

C. $(x - 2)^2 + (y + 6)^2 = 34$

D. $(x + 2)^2 + (y - 6)^2 = 34$

18) What is the directrix of the parabola represented by $(y + 4)^2 = 8(x - 5)$?

A. $x = 5$

B. $x = 3$

C. $y = 5$

D. $y = 3$

19) Find the center and radius of the circle with equation $x^2 - 4x + y^2 + 6y = 23$.

A. Center: (−2,3); radius: 6

B. Center: (2,−3); radius: 6

C. Center: (−2,3); radius: 36

D. Center: (2,−3); radius: 36

20) Given these 3 quadratic functions, where a and k are positive integers and $k \neq 1$:

$$y = x^2 - 2ax + a$$

$$y = x^2 - 2ax + a + k$$

$$y = x^2 - 2ax + a + k^2$$

What must be true about their minimum points?

I. The minimum points have the same x-coordinate.

II. The minimum points have the same y-coordinate.

III. The minimum points have different x-coordinates.

IV. The minimum points have different y-coordinates.

A. I and II only

B. I and IV only

C. II and III only

D. III and IV only

21) Circle A is externally tangent to the parabola $y - 1 = -(x - 3)^2$ and is symmetric about the line $x = 3$. The diameter of the circle is 2 units. What is the equation of the circle?

A. $(x - 3)^2 + (y - 1)^2 = 4$

B. $(x - 1)^2 + (y - 4)^2 = 2$

C. $(x - 3)^2 + (y - 2)^2 = 1$

D. $(x - 1)^2 + (y - 3)^2 = 4$

22) Which equation represents a parabola with directrix $x = 6$ and focus $(2,-3)$?

A. $x = -\frac{1}{8}(y + 3)^2 + 4$

B. $x = -\frac{1}{16}(y + 3)^2 + 2$

C. $y = -\frac{1}{8}(x - 2)^2 + 4\frac{1}{2}$

D. $y = -\frac{1}{6}(x - 2)^2 + 4\frac{1}{2}$

23) What is the standard form of the equation for a circle with center $(2,-3)$ and area of 16π?

A. $(x-2)^2+(y+3)^2=4$

B. $(x+2)^2+(y-3)^2=4$

C. $(x-2)^2+(y+3)^2=16$

D. $(x+2)^2+(y-3)^2=16$

24) A circle is tangent to the x-axis at $(-4,0)$ and the y-axis at $(0,4)$. What is the equation of this circle?

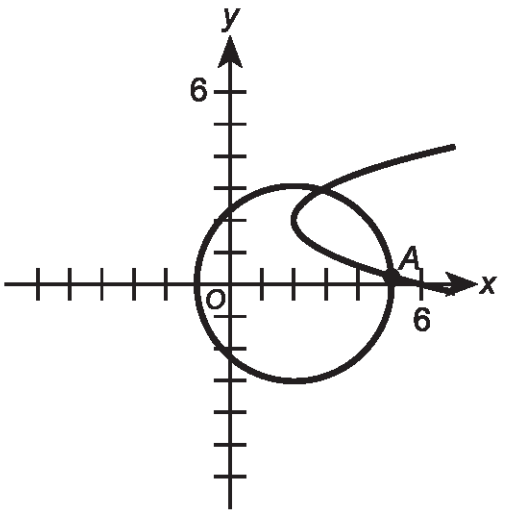
A. $(x-4)^2+(y+4)^2=4$

B. $(x+4)^2+(y-4)^2=4$

C. $(x-4)^2+(y+4)^2=16$

D. $(x+4)^2+(y-4)^2=16$

25) Which of the following systems of equations could be used to find the point of intersection labeled *A* in this graph?



A. $\begin{cases} y = \sqrt{9-(x-2)^2} \\ y = 2+\sqrt{x-2} \end{cases}$

B. $\begin{cases} y = \sqrt{9-(x-2)^2} \\ y = 2-\sqrt{x-2} \end{cases}$

C. $\begin{cases} y = -\sqrt{9-(x-2)^2} \\ y = 2+\sqrt{x-2} \end{cases}$

D. $\begin{cases} y = -\sqrt{9-(x-2)^2} \\ y = 2-\sqrt{x-2} \end{cases}$

26) In the standard (x,y) coordinate plane, circle Q has center $Q(3,3)$ and passes through the origin. Circle Q' is found by reflecting circle Q into Quadrant III over the line $y = -x$. What is the equation of circle Q'?

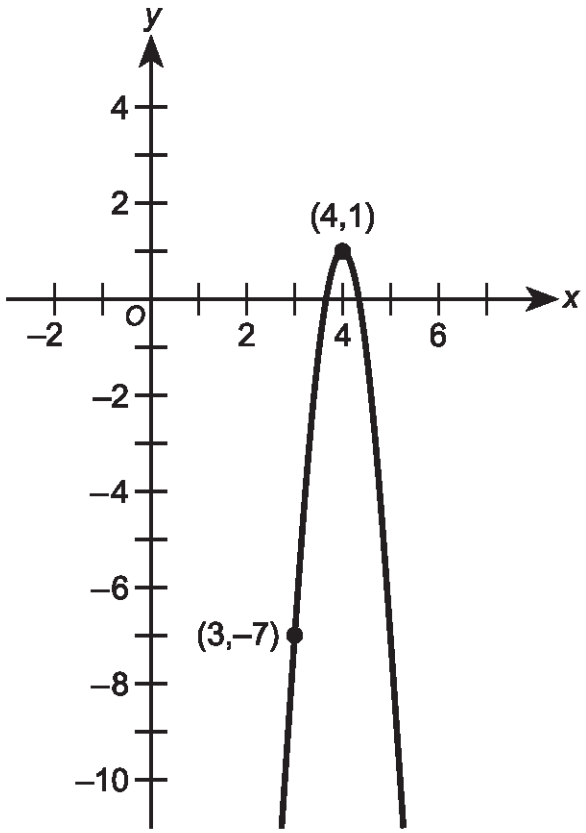
A. $(x+3)^2 + (y+3)^2 = 9$

B. $(x+3)^2 + (y-3)^2 = 18$

C. $(x-3)^2 + (y+3)^2 = 9$

D. $(x+3)^2 + (y+3)^2 = 18$

27) Let $y = f(x)$ be a parabola with vertex (4,1) that passes through the point (3,−7).



A. What are the domain and range of $f(x)$? Explain how you found your answers.

B. What is the equation of this parabola in vertex form? Show your work algebraically. Explain how you used the points indicated on the graph to determine the equation.

C. Two transformations are performed on the graph of $f(x)$ to create a new function, $g(x)$. First, the graph is reflected over the x-axis. What single additional transformation is needed so that $g(x)$ has exactly 1 real root? Explain how you know that $g(x)$ has exactly 1 real root.

28) Consider the function $g(x) = -(x+3)^2 + 1$.

A. Describe the transformations that could occur to the graph of $f(x) = x^2$ to get $g(x)$.

B. Graph $g(x)$ using the vertex and at least 3 points on one side of the vertex. Show your work algebraically and label the points using ordered pairs.

C. What are the domain and range of $g(x)$? Explain how you determined your answers.

29) Consider the circle with the equation $(x - 2)^2 + (y + 3)^2 = 64$.

A. What are the center and radius of the circle? Explain how you determined your answer.

B. Sandy says $(-2,-10)$ is a point on the circle. Is Sandy correct? Show your work algebraically, and explain how you made your decision.

C. Graph the circle. Explain the procedure you used to graph the circle.

30) The general form of a particular circle is $x^2 + y^2 - 4x + 2y - 11 = 0$.

A. What is the standard form of the equation of the circle? Show your work algebraically, and explain the approach you used to find your answer.

B. Determine the coordinates for the center of the circle. Explain how you determined your answer.

C. Determine the x-coordinates of each point on the circle with a y-coordinate 2 less than the center's y-coordinate. Simplify your answer completely. Show your algebraic work, and explain the approach you used to find your answer.

Please use the space below to write your response(s) to the writing assignment provided by your teacher. If there are multiple tasks to the question, please clearly label the number or letter of each task in the column to the left of your answers. If you need additional pages for your response, your teacher can provide them.

Please write the name of the writing assignment here: _____

Task

QualityCore™

Reference Sheet for the QualityCore™ **Algebra II End-of-Course Assessment**

Equations of a Line

Standard Form	$Ax + By = C$	A, B, and C are constants with A and B not both equal to zero.
Slope-Intercept Form	$y = mx + b$	(x_1,y_1) is a point. m = slope b = y-intercept
Point-Slope Form	$y - y_1 = m(x - x_1)$	

Quadratics

Standard Form of a Quadratic Equation	$ax^2 + bx + c = 0$	a, b, and c are constants, where $a \neq 0$.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Conic Sections

Circle	$(x - h)^2 + (y - k)^2 = r^2$	center (h,k) r = radius	
Parabola	$y = a(x - h)^2 + k$	axis of symmetry $x = h$ directrix $y = k - \frac{1}{4a}$	vertex (h,k) focus $\left(h, k + \frac{1}{4a}\right)$
Parabola	$x = a(y - k)^2 + h$	axis of symmetry $y = k$ directrix $x = h - \frac{1}{4a}$	vertex (h,k) focus $\left(h + \frac{1}{4a}, k\right)$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$, center (h,k)	
Ellipse	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 - b^2$, center (h,k)	
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$, center (h,k)	
Hyperbola	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 + b^2$, center (h,k)	

Lines and Points

Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	(x_1,y_1) and (x_2,y_2) are 2 points. m = slope M = midpoint d = distance
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	
Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	

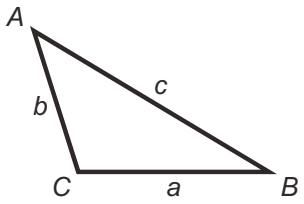
continued

Miscellaneous

Distance, Rate, Time	$D = rt$	D = distance r = rate t = time I = interest p = principal A = amount of money after t years n = number of times interest is compounded annually
Simple Interest	$I = prt$	
Compound Interest	$A = p\left(1 + \frac{r}{n}\right)^{nt}$	
Pythagorean Theorem	$a^2 + b^2 = c^2$	a and b = legs of right triangle c = hypotenuse

Laws of Sines and Cosines

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$



Sequences, Series, and Counting

Arithmetic Sequence	$a_n = a_1 + (n - 1)d$	$a_n = n^{\text{th}}$ term n = number of the term d = common difference s_n = sum of the first n terms r = common ratio k = number of objects in the set m = number of objects selected
Arithmetic Series	$s_n = \frac{n}{2}(a_1 + a_n)$	
Geometric Sequence	$a_n = a_1(r^{n-1})$	
Geometric Series	$s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$	
Combinations	${}_kC_m = C(k,m) = \frac{k!}{(k-m)!\,m!}$	
Permutations	${}_kP_m = P(k,m) = \frac{k!}{(k-m)!}$	

Circumference, Area, and Volume

Triangle	$A = \frac{1}{2}bh$	A = area b = base h = height r = radius C = circumference d = diameter V = volume B = area of base $\pi \approx 3.14$
Parallelogram	$A = bh$	
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	
Circle	$A = \pi r^2$ $C = \pi d$	
General Prism	$V = Bh$	
Right Circular Cylinder	$V = \pi r^2 h$	
Pyramid	$V = \frac{1}{3}Bh$	
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	
Sphere	$V = \frac{4}{3}\pi r^3$	

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Answer Key

1) D
2) A
3) D
4) A
5) C
6) B
7) B
8) C
9) A
10) C
11) D
12) C
13) B
14) D
15) A
16) D
17) A
18) B
19) B
20) B
21) C
22) A
23) C
24) D
25) B
26) D

Scoring Criteria:

27) A 4-point response will include, but is not limited to, the following points:

A. Domain of the graph of the parabola: All real numbers or $(-\infty, \infty)$

Explanation of how the domain was found: This function is defined for all real numbers. A quadratic function does not involve roots or rational expressions with variables in the denominator; therefore, there are no restrictions on the value for x. This makes the domain of $f(x)$ all real numbers.

OR

Looking at the graph, one can see that the parabola continues to expand horizontally, suggesting that any value of x will have a corresponding y-value. Therefore, x can be any real number.

Range of the graph of the parabola: All real numbers less than or equal to 1, $y \leq 1$, or $(-\infty, 1]$

Explanation of how the range was found: Since this parabola opens downward, the vertex, (4,1), is a maximum for the graph. Therefore, the range is all real numbers equal to or less than 1.

OR

Looking at the graph, one can see that all y-coordinates for points on the graph are less than or equal to 1.

B. Correct equation in vertex form: $y = -8(x - 4)^2 + 1$

Appropriate algebraic work:

$$y = a(x-h)^2 + k$$
$$y = a(x-4)^2 + 1$$
$$-7 = a(3 - 4)^2 + 1$$
$$-7 = a(-1)^2 + 1$$
$$-7 = a + 1$$
$$-8 = a$$

Using substitution, the equation becomes: $y = -8(x-4)^2 + 1$

Explanation of how the points were used to determine the equation: The vertex (4,1) and a point on the parabola (3,–7) are given. Unknowns h and k in the vertex form for the equation of the parabola correspond to the coordinates of the vertex of the parabola. Since the vertex is identified on the graph, I know $h = 4$ and $k = 1$. The graph itself represents solutions to the equation, which means that the additional point identified on the graph is a solution. Therefore, I can substitute 3 for x and –7 for y. Given the values for h, k, x, and y, I can substitute them into the equation for the parabola and solve for a. Finally, I use the value a, h, k, to write the equation for the parabola in vertex form.

Note: A student could also use symmetry of parabola to locate the point (5,–7). Using the 3 known points (4,1), (3,–7), and (5,–7), the students could write 3 equations with 3 unknowns (*a*, *h*, and *k*). This system could then be solved for the unknowns, allowing the students to write the equation of the parabola.

C. Additional transformation: Shift up 1 unit

Explanation of how the answer was found: After reflecting $f(x)$ across the x-axis, the graph would be a parabola that opens up with a vertex at (4,–1). This function has 2 zeros (or roots) because the graph intersects the x-axis at 2 different points. Since $g(x)$ is supposed to be a quadratic function with 1 real root, the graph can only intersect the x-axis at 1 point. This means that the vertex must be on the x-axis. If you shift the entire function up 1 unit, the vertex will be (4,0), which is on the x-axis. Therefore, to obtain $g(x)$ after the reflection, you have to shift the function up 1 unit.

Note: Although not required, a student may also include sketches of the transformations to aid in the explanation.

Note: Since $g(x)$ is identified as a function in the task, transformations that lead to a horizontally oriented parabola are not appropriate for this portion of the response.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

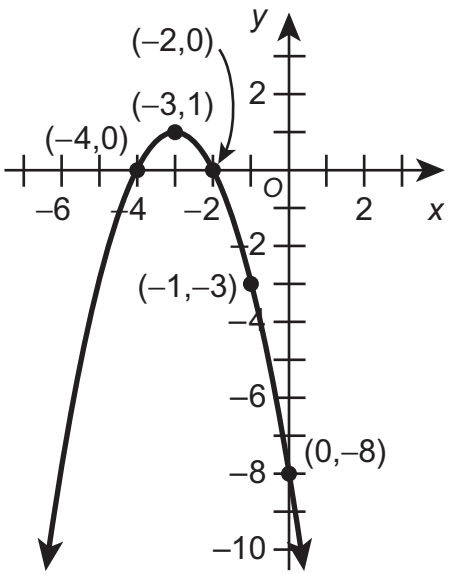
Scoring Criteria:

28)

A 4-point response may include, but is not limited to, the following points:

A. **Description of the transformations that could occur to the graph of $f(x) = x^2$ to get $g(x)$:** I shifted the graph left 3 units. I reflected the graph across the *x*-axis. Then, I shifted the graph up 1 unit.

B. **Correct graph:**



Appropriate work needed to find the answer:

x	*y*
–6	–8
–5	–3
–4	0
–3	1
–2	0
–1	–3
0	–8

$g(-6) = -(-6+3)^2 + 1 = -(-3)^2 + 1 = -9 + 1 = -8$
$g(-5) = -(-5+3)^2 + 1 = -(-2)^2 + 1 = -4 + 1 = -3$

$g(-4) = -(-4+3)^2 + 1 = -(-1)^2 + 1 = -1 + 1 = 0$
$g(-3) = -(-3+3)^2 + 1 = -(0)^2 + 1 = 0 + 1 = 1$
$g(-2) = -(-2+3)^2 + 1 = -(1)^2 + 1 = -1 + 1 = 0$
$g(-1) = -(-1+3)^2 + 1 = -(2)^2 + 1 = -4 + 1 = -3$
$g(0) = -(0+3)^2 + 1 = -(3)^2 + 1 = -9 + 1 = -8$

C. **Correct domain of *g*(*x*):** All real numbers
Correct range of *g*(*x*): $y \leq 1$

Explanation of how the answers were determined: The domain is all real numbers because any number can be substituted in for *x* in $g(x)$. The range is $y \leq 1$ because $g(x)$ is a parabola with the vertex at (−3,1). Since the parabola opens downward, the range will be all numbers less than or equal to the *y*-value of the vertex.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

Scoring Criteria:

29) A 4-point response may include, but is not limited to, the following points:

A. **Correct center:** (2,−3)

Correct radius: 8

Explanation of how the answer was determined: To find the center, I took the expressions inside each set of parentheses and solved for the variable ($x-2=0 \Rightarrow x=2$ and $y+3=0 \Rightarrow y=-3$). To find the radius, I took the square root of 64.

OR

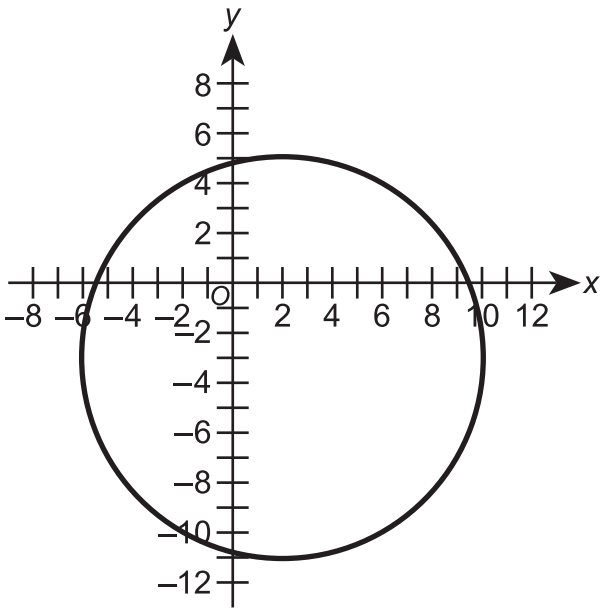
The standard form of an equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (*h*,*k*) is the center and *r* is the radius.

B. **Determination if Sandy was correct:** No, the point is not on the circle.

Appropriate work needed to make the decision:
$(-2-2)^2+(-10+3)^2=(-4)^2+(-7)^2=16+49=65;\ 65\neq 64$

Explanation of how your decision was made: I substituted −2 for *x* and −10 for *y* in the left-hand side of the equation for the circle. I evaluated the expression and obtained 65. Since 65 is not equal to 64, then the point (−2,−10) is not on the circle.

C. **Correct graph:**



Explanation of the procedure used to graph the circle: I graphed the center of the circle at (2,−3). I then used the radius to find 4 points on the circle by moving up 8 units to (2,5), down 8 units to (2,−11), right 8 units to (10,−3), and left 8 units to (−6,−3). Then, I connected the points to form a circle.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

Scoring Criteria:

30)

A 4-point response may include, but is not limited to, the following points:

A. **Correct equation:** $(x-2)^2+(y+1)^2=16$

Appropriate work needed to find the answer:

$x^2-4x+y^2+2y-11=0$

$x^2-4x+y^2+2y=11$

$x:\left(\frac{-4}{2}\right)^2=(-2)^2=4$

$y:\left(\frac{2}{2}\right)^2=(1)^2=1$

$x^2-4x+4+y^2+2y+1=11+4+1$

$(x-2)^2+(y+1)^2=16$

Explanation of the approach used to find the answer: I collected the *x* terms and the *y* terms. I added 11 to both sides of the equation, and completed the square for *x* and *y*. I used the following steps: I took half of the coefficient on the linear term. I squared those numbers and added both of them to both sides of the equation. I wrote the expression on the left-hand side as 2 perfect square trinomials.

B. **Correct center:** (2,−1)

Explanation of how the answer was determined: I took the expressions inside each set of parentheses and solved for the variable to find each coordinate ($x-2=0\Rightarrow x=2$ and $y+1=0\Rightarrow y=-1$).

OR

The standard form of an equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where (*h*,*k*) is the center.

C. **Correct *x*-coordinates:** $x=2\pm2\sqrt{3}$

Appropriate work needed to find the answer:

$-1-2=-3$

$(x-2)^2+(-3+1)^2=16$

$(x-2)^2+(-2)^2=16$

$(x-2)^2+4=16$

$(x-2)^2=12$

$x-2=\pm\sqrt{12}$

$x-2=\pm2\sqrt{3}$

Explanation of the approach used to find the answer: I subtracted 2 from −1 to get −3 to get the new *y*-coordinate. I substituted −3 for *y* in the standard-form equation. I

simplified the expression. I took the square root of both sides of the equation, simplified the radical, and added 2 to both sides of the equation.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

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