Name: Date:
Teacher: Class/Period:

1) Which function has the same range as $y = (x + 3)^2$?

A. $y = (x + 3)^2 - 2$

B. $y = x^2 + 9$

C. $y = 2(x - 3)^2 + 1$

D. $y = (x - 5)^2$

2) Determine the domain and range of
$y = 2x^2 + 2x - 4$.

A. Domain: $-2 \le x \le 1$
Range: $y \ge -\frac{9}{2}$ and $y \le \frac{9}{2}$

B. Domain: all real numbers
Range: $-2 \le x \le 1$

C. Domain: all real numbers
Range: $y \le -\frac{9}{2}$

D. Domain: all real numbers
Range: $y \ge -\frac{9}{2}$

3) What is the range of $y = 3x^2 - 2x + 5$?

A. $[\frac{5}{3}, \infty)$

B. $[\frac{14}{3}, \infty)$

C. $(-\infty, \frac{14}{3}]$

D. $(-\infty, \frac{5}{3}]$

4) Given $f(x) = -ax^2 + 2ahx - ah^2 + k$, find the domain and range of $f(x)$ where $a > 0$, $h > 0$, and $k > 0$.

A. Domain: $(-\infty, \infty)$
Range: $(-\infty, k]$

B. Domain: $(a(x - h)^2, \infty)$
Range: $(-\infty, k]$

C. Domain: $(a(x - h)^2, \infty)$
Range: $[k, \infty)$

D. Domain: $(-\infty, \infty)$
Range: $(\infty, k]$

5) Which transformations can be performed on the graph of $f(x) = x^2$ that result in the graph of $f'(x) = -2x^2 - 12x - 13$?

A. Shift left 3 units, stretch horizontally by a factor of 2, reflect through the y-axis, and shift down 5 units

B. Shift right 3 units, stretch horizontally by a factor of 2, reflect through the y-axis, and shift down 5 units

C. Shift left 3 units, stretch vertically by a factor of 2, reflect through the x-axis, and shift up 5 units

D. Shift right 3 units, stretch vertically by a factor of 2, reflect through the x-axis, and shift down 5 units

6) The function $y = (x + 2)^2 + 3$ is reflected across the y-axis. What are the coordinates of the vertex after this reflection?

A. $(-2,-3)$

B. $(-2,3)$

C. $(2,-3)$

D. $(2,3)$

7) Which equation is the reflection of $y = x^2 - 4x + 3$ across the x-axis?

A. $y = x^2 - 4x + 3$

B. $y = x^2 - 4x - 3$

C. $y = -x^2 + 4x - 3$

D. $y = -x^2 + 4x + 3$

8) A certain relation is defined by these ordered pairs:

$$\{(2,2), (-2,2), (2,-2), (-2,-2)\}$$

If this relation is translated 5 units to the right, what is the resulting relation?

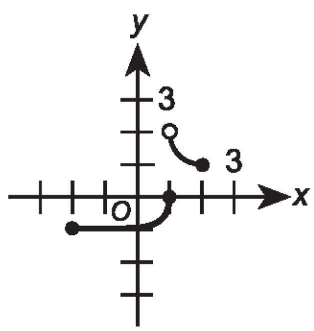
A. $\{(2,7), (-2,7), (2,3), (-2,3)\}$

B. $\{(7,7), (5,7), (7,3), (3,3)\}$

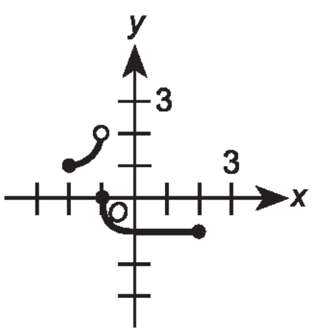
C. $\{(10,2), (-10,2), (10,-2), (-10,-2)\}$

D. $\{(7,2), (3,2), (7,-2), (3,-2)\}$

9) This graph shows $y = f(x)$. What is the graph of $y = -f(x - 1)$?



A.



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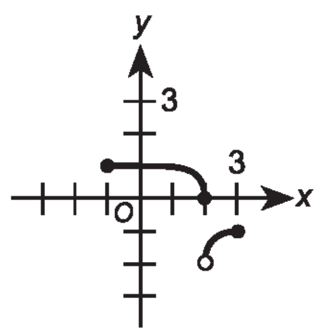

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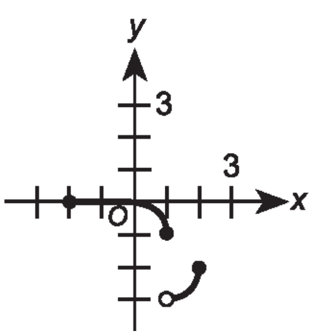

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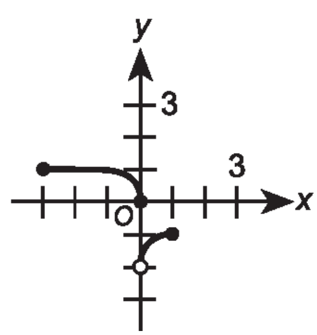


10) If $f(x) = |x - 1| - 2$, what is the vertex of $y = f(x + 2) - 1$?

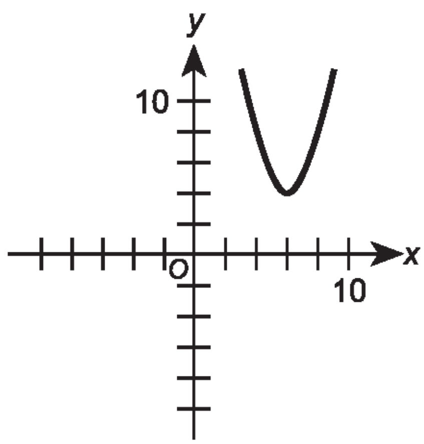
A. $(-2,-1)$

B. $(-1,-3)$

C. $(2,-1)$

D. $(3,-3)$

11) This graph of the function $f(x)$ is rotated 90° counterclockwise about the vertex and then shifted down 2 units and to the left 7 units. Which equation will describe the new graph?



A. $x = -(y - 6)^2 + 13$

B. $x = -(y - 2)^2 - 1$

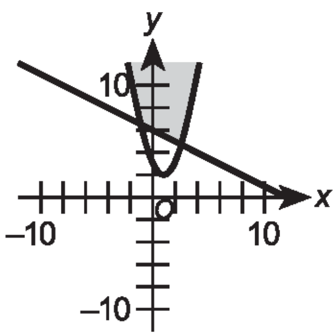
C. $x = (y - 2)^2 - 13$

D. $x = (y - 2)^2 - 1$

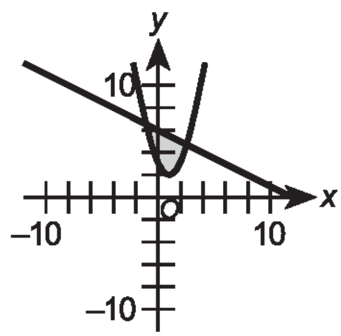
12) Which graph represents the solution set of this system of inequalities?

$$\begin{cases} y \geq x^2 - 2x + 3 \\ x + 2y \leq 12 \end{cases}$$

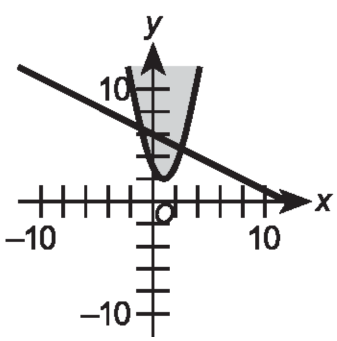
A.



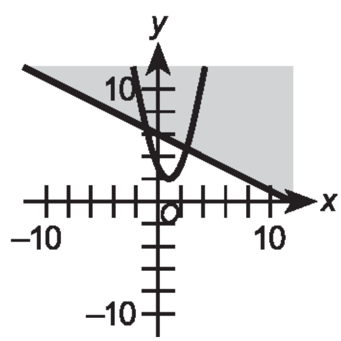
C.



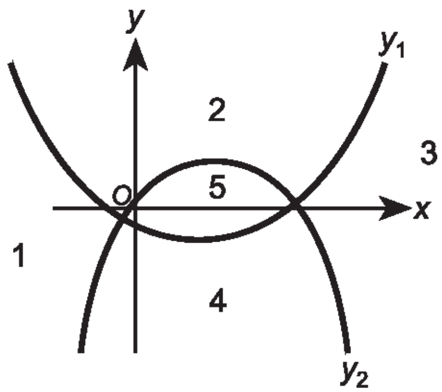
B.



D.



13) This graph shows $y_1 = x^2 - 3x - 5$ and $y_2 = -x^2 + 4x + 1$.



What region(s) should be shaded to represent this system of inequalities?

$$\begin{cases} y_1 \geq x^2 - 3x - 5 \\ y_2 \geq -x^2 + 4x + 1 \end{cases}$$

A. 2

B. 5

C. 2 and 3

D. 2 and 5

14) A parabola has vertex (2,3), focus (2,7), and directrix $y = -1$. What is the equation of the parablola?

A. $(y - 2)^2 = 16(x - 3)$

B. $(y + 2)^2 = 16(x + 3)$

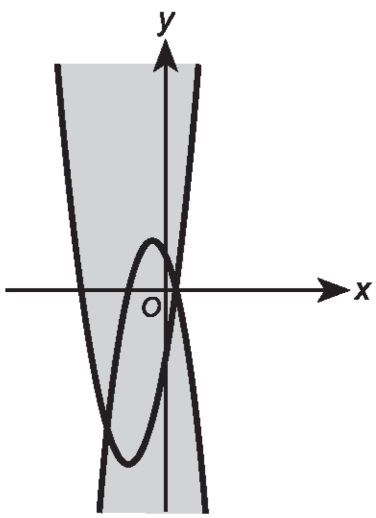
C. $(x - 2)^2 = 16(y - 3)$

D. $(x + 2)^2 = 16(y + 3)$

15) Which graph represents the solution set to this system of inequalities?

$$\begin{cases} y \geq x^2 + 6x - 5 \\ y \leq -x^2 - 2x + 3 \end{cases}$$

A.



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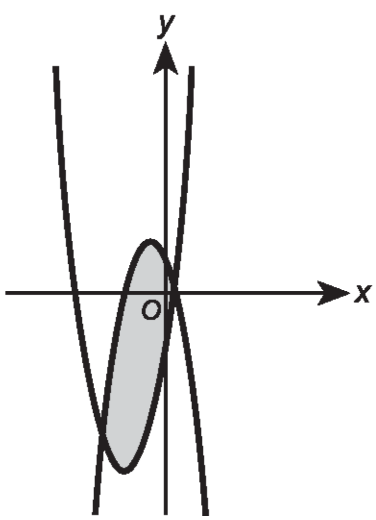

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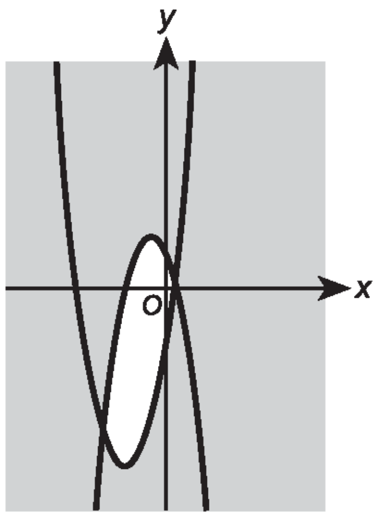

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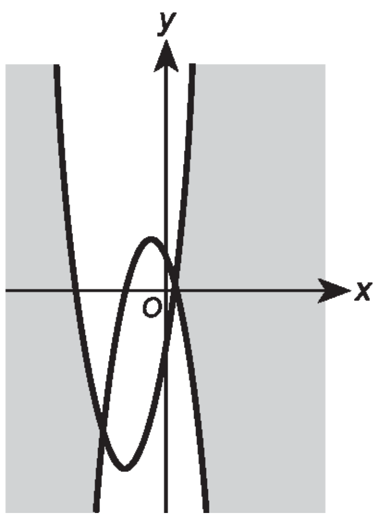
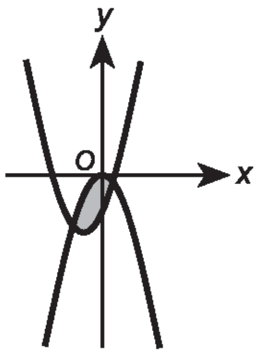


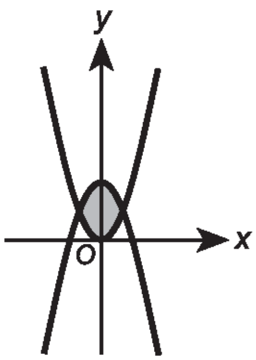
16) Which is the graphical representation of the solution set for this system of inequalities?

$$\begin{cases} y \geq x^2 \\ y \leq -(x+1)^2+3 \end{cases}$$

A.



B.



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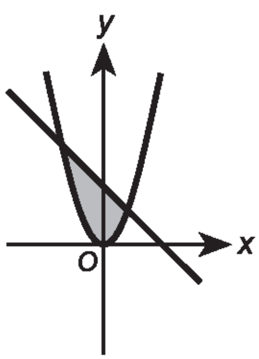

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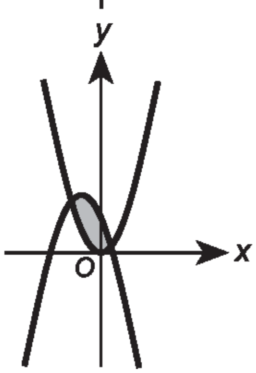
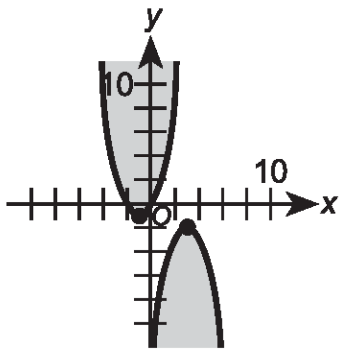


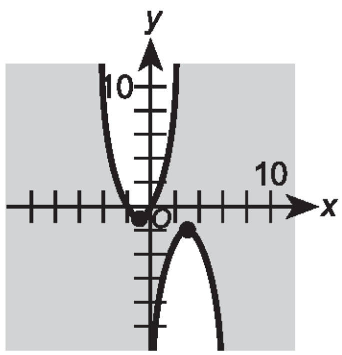
17) Which graph represents the solution set to this system of quadratic inequalities?

$$\begin{cases} y \geq -(x-3)^2 - 2 \\ y \leq (x+1)^2 - 1 \end{cases}$$

A.



B.



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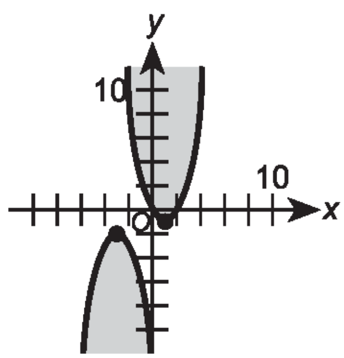
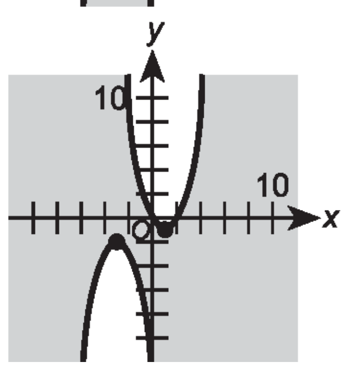
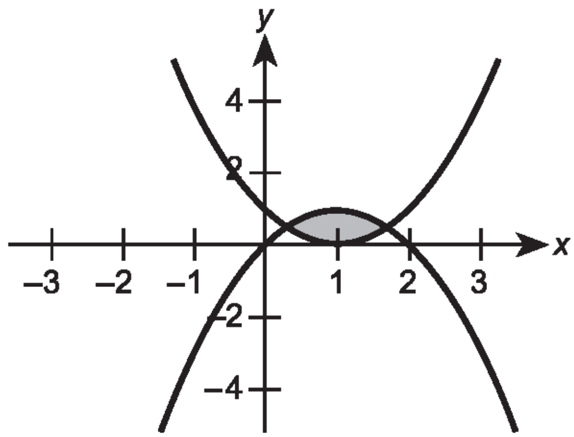


D.



18) Which system of inequalities describes the shaded region in this graph?



A. $0.5 \le x \le 1.5$ and $0 \le y \le 1$

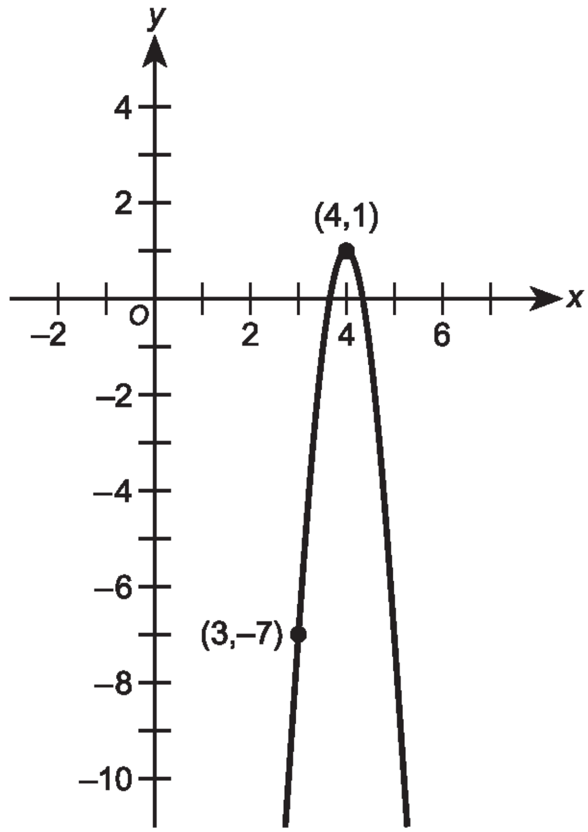
B. $y \ge x^2$ and $y \le -x^2$

C. $x^2 \le y \le -x^2 + 1$

D. $y \ge (x - 1)^2$ and $y \le -(x - 1)^2 + 1$

19) Determine the domain and range of the function $y = -3(x + 7)^2 - 8$. Explain how the vertex of a parabola helps determine the range of the function.

20) Let $y = f(x)$ be a parabola with vertex (4,1) that passes through the point (3,−7).



A. What are the domain and range of $f(x)$? Explain how you found your answers.

B. What is the equation of this parabola in vertex form? Show your work algebraically. Explain how you used the points indicated on the graph to determine the equation.

C. Two transformations are performed on the graph of $f(x)$ to create a new function, $g(x)$. First, the graph is reflected over the x-axis. What single additional transformation is needed so that $g(x)$ has exactly 1 real root? Explain how you know that $g(x)$ has exactly 1 real root.

21) Consider the function $g(x) = -(x+3)^2 + 1$.

A. Describe the transformations that could occur to the graph of $f(x) = x^2$ to get $g(x)$.

B. Graph $g(x)$ using the vertex and at least 3 points on one side of the vertex. Show your work algebraically and label the points using ordered pairs.

C. What are the domain and range of $g(x)$? Explain how you determined your answers.

22) Consider the real-valued functions $f(x) = x^2 - 6$ and $g(x) = 2x - 3$.

A. What are the domain and range of each function? Explain how you determined your answers.

B. Find $f(g(x))$. Show your algebraic work, and explain the approach you used to find your answer.

C. What are the domain and range of $f(g(x))$? Show your algebraic work, and explain the approach you used to find your answer.

23) Given the system of inequalities shown:

$$\begin{cases} y \geq (x-2)^2 \\ y < -x^2 + 5 \end{cases}$$

A. Graph the system of inequalities. Show the work you used to obtain at least 3 points on the boundary equation of each graph and to decide the direction of the shading for each boundary equation.

B. Find the exact points of intersection for each boundary equation in the system of inequalities. Show your work algebraically, and explain the approach you used to find your answer.

24) Graph $y = 3 - 2e^{x-1}$. Explain how you graphed the function using transformations of the graph of $y = e^x$. Show algebraic work for obtaining the exact coordinates of at least 3 points on the graph.

Please use the space below to write your response(s) to the writing assignment provided by your teacher. If there are multiple tasks to the question, please clearly label the number or letter of each task in the column to the left of your answers. If you need additional pages for your response, your teacher can provide them.

Please write the name of the writing assignment here: ______________________________________

Task

QualityCore™

Reference Sheet for the QualityCore™
Algebra II End-of-Course Assessment

Equations of a Line

Standard Form	$Ax + By = C$	A, B, and C are constants with A and B not both equal to zero.
Slope-Intercept Form	$y = mx + b$	(x_1, y_1) is a point.
Point-Slope Form	$y - y_1 = m(x - x_1)$	m = slope b = y-intercept

Quadratics

Standard Form of a Quadratic Equation	$ax^2 + bx + c = 0$	a, b, and c are constants, where $a \neq 0$.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Conic Sections

Circle	$(x - h)^2 + (y - k)^2 = r^2$	center (h,k) r = radius	
Parabola	$y = a(x - h)^2 + k$	axis of symmetry $x = h$ directrix $y = k - \frac{1}{4a}$	vertex (h,k) focus $\left(h, k + \frac{1}{4a}\right)$
Parabola	$x = a(y - k)^2 + h$	axis of symmetry $y = k$ directrix $x = h - \frac{1}{4a}$	vertex (h,k) focus $(h + \frac{1}{4a}, k)$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$, center (h,k)	
Ellipse	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 - b^2$, center (h,k)	
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$, center (h,k)	
Hyperbola	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	foci $(h, k \pm c)$ where $c^2 = a^2 + b^2$, center (h,k)	

Lines and Points

Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	(x_1,y_1) and (x_2,y_2) are 2 points. m = slope
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	M = midpoint d = distance
Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	

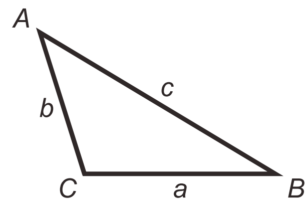
continued

Miscellaneous

Distance, Rate, Time	$D = rt$	D = distance
Simple Interest	$I = prt$	r = rate
Compound Interest	$A = p\left(1 + \frac{r}{n}\right)^{nt}$	t = time
		I = interest
		p = principal
		A = amount of money after t years
		n = number of times interest is compounded annually
Pythagorean Theorem	$a^2 + b^2 = c^2$	a and b = legs of right triangle
		c = hypotenuse

Laws of Sines and Cosines

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$	



Sequences, Series, and Counting

Arithmetic Sequence	$a_n = a_1 + (n - 1)d$	a_n = n^{th} term
Arithmetic Series	$s_n = \frac{n}{2}(a_1 + a_n)$	n = number of the term
Geometric Sequence	$a_n = a_1(r^{n-1})$	d = common difference
Geometric Series	$s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$	s_n = sum of the first n terms
Combinations	${}_kC_m = C(k,m) = \frac{k!}{(k-m)!\,m!}$	r = common ratio
Permutations	${}_kP_m = P(k,m) = \frac{k!}{(k-m)!}$	k = number of objects in the set
		m = number of objects selected

Circumference, Area, and Volume

Triangle	$A = \frac{1}{2}bh$	A = area
Parallelogram	$A = bh$	b = base
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	h = height
Circle	$A = \pi r^2$ $C = \pi d$	r = radius
		C = circumference
		d = diameter
General Prism	$V = Bh$	V = volume
Right Circular Cylinder	$V = \pi r^2 h$	B = area of base
Pyramid	$V = \frac{1}{3}Bh$	$\pi \approx 3.14$
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	
Sphere	$V = \frac{4}{3}\pi r^3$	

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Answer Key

1) D
2) D
3) B
4) A
5) C
6) D
7) C
8) D
9) C
10) B
11) B
12) C
13) A
14) C
15) B
16) D
17) B
18) D

Answer:

19) The domain is all real numbers. The range is all numbers less than or equal to −8.

The graph of a quadratic function is a parabola. The range of a quadratic function with vertex (h,k) is either $\{y \mid y \le k\}$ if the parabola opens down, or $\{y \mid y \ge k\}$ if the parabola opens up.

Scoring Criteria:

20) A 4-point response will include, but is not limited to, the following points:

A. Domain of the graph of the parabola: All real numbers or $(-\infty, \infty)$

Explanation of how the domain was found: This function is defined for all real numbers. A quadratic function does not involve roots or rational expressions with variables in the denominator; therefore, there are no restrictions on the value for *x*. This makes the domain of $f(x)$ all real numbers.

OR

Looking at the graph, one can see that the parabola continues to expand horizontally, suggesting that any value of *x* will have a corresponding *y*-value. Therefore, *x* can be any real number.

Range of the graph of the parabola: All real numbers less than or equal to 1, $y \leq 1$, or $(-\infty, 1]$

Explanation of how the range was found: Since this parabola opens downward, the vertex, (4,1), is a maximum for the graph. Therefore, the range is all real numbers equal to or less than 1.

OR

Looking at the graph, one can see that all *y*-coordinates for points on the graph are less than or equal to 1.

B. Correct equation in vertex form: $y = -8(x - 4)^2 + 1$

Appropriate algebraic work:

$$y = a(x-h)^2 + k$$
$$y = a(x-4)^2 + 1$$
$$-7 = a(3 - 4)^2 + 1$$
$$-7 = a(-1)^2 + 1$$
$$-7 = a + 1$$
$$-8 = a$$

Using substitution, the equation becomes: $y = -8(x-4)^2 + 1$

Explanation of how the points were used to determine the equation: The vertex (4,1) and a point on the parabola (3,–7) are given. Unknowns *h* and *k* in the vertex form for the equation of the parabola correspond to the coordinates of the vertex of the parabola. Since the vertex is identified on the graph, I know $h = 4$ and $k = 1$. The graph itself represents solutions to the equation, which means that the additional point identified on the graph is a solution. Therefore, I can substitute 3 for *x* and –7 for y. Given the values for *h*, *k*, *x*, and *y*, I can substitute them into the equation for the parabola and solve for *a*. Finally, I use the value *a*, *h*, *k*, to write the equation for the parabola in vertex form.

Note: A student could also use symmetry of parabola to locate the point (5,–7). Using the 3 known points (4,1), (3,–7), and (5,–7), the students could write 3 equations with 3 unknowns (a, h, and k). This system could then be solved for the unknowns, allowing the students to write the equation of the parabola.

C. Additional transformation: Shift up 1 unit

Explanation of how the answer was found: After reflecting $f(x)$ across the x-axis, the graph would be a parabola that opens up with a vertex at (4,–1). This function has 2 zeros (or roots) because the graph intersects the x-axis at 2 different points. Since $g(x)$ is supposed to be a quadratic function with 1 real root, the graph can only intersect the x-axis at 1 point. This means that the vertex must be on the x-axis. If you shift the entire function up 1 unit, the verte*x* will be (4,0), which is on the x-axis. Therefore, to obtain $g(x)$ after the reflection, you have to shift the function up 1 unit.

Note: Although not required, a student may also include sketches of the transformations to aid in the explanation.

Note: Since $g(x)$ is identified as a function in the task, transformations that lead to a horizontally oriented parabola are not appropriate for this portion of the response.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

1 **A response at this level provides evidence of minimal knowledge and understanding of the subject matter.**

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
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- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

0 **A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

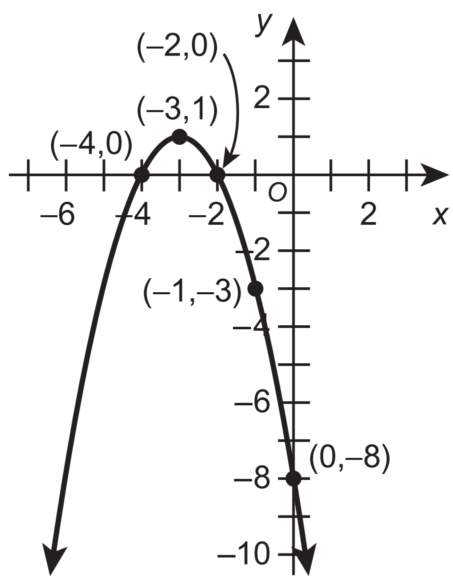
Scoring Criteria:

21)

A 4-point response may include, but is not limited to, the following points:

A. **Description of the transformations that could occur to the graph of $f(x) = x^2$ to get $g(x)$:** I shifted the graph left 3 units. I reflected the graph across the x-axis. Then, I shifted the graph up 1 unit.

B. **Correct graph:**



Appropriate work needed to find the answer:

x	y
–6	–8
–5	–3
–4	0
–3	1
–2	0
–1	–3
0	–8

$g(-6) = -(-6+3)^2 + 1 = -(-3)^2 + 1 = -9 + 1 = -8$

$g(-5) = -(-5+3)^2 + 1 = -(-2)^2 + 1 = -4 + 1 = -3$

$g(-4) = -(-4+3)^2 + 1 = -(-1)^2 + 1 = -1 + 1 = 0$

$g(-3) = -(-3+3)^2 + 1 = -(0)^2 + 1 = 0 + 1 = 1$

$g(-2) = -(-2+3)^2 + 1 = -(1)^2 + 1 = -1 + 1 = 0$

$g(-1) = -(-1+3)^2 + 1 = -(2)^2 + 1 = -4 + 1 = -3$

$g(0) = -(0+3)^2 + 1 = -(3)^2 + 1 = -9 + 1 = -8$

C. **Correct domain of *g*(*x*):** All real numbers
Correct range of *g*(*x*): $y \leq 1$

Explanation of how the answers were determined: The domain is all real numbers because any number can be substituted in for x in $g(x)$. The range is $y \leq 1$ because $g(x)$ is a parabola with the vertex at (−3,1). Since the parabola opens downward, the range will be all numbers less than or equal to the *y*-value of the vertex.

Rubric:

4 **A response at this level provides evidence of thorough knowledge and understanding of the subject matter.**

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.

3 **A response at this level provides evidence of competent knowledge and understanding of the subject matter.**

- The response addresses most parts of the question or problem correctly.
- The response includes some minor errors but generally uses appropriate procedures accurately.
- The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
- The explanation in the response is mostly clear and coherent.

2 **A response at this level provides evidence of a basic knowledge and understanding of the subject matter.**

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

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- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
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- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.

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Scoring Criteria:

22) A 4-point response may include, but is not limited to, the following points:

A. **Correct domain for *f*(*x*):** All real numbers

Correct range for *f*(*x*): $y \geq -6$

Correct domain for *g*(*x*): All real numbers

Correct range for *g*(*x*): All real numbers

Explanation of how the answer was determined: The domain is all real numbers because any number can be substituted for *x* in *f*(*x*). The range is $y \geq -6$ because *f*(*x*) is a parabola with the vertex at (0,−6). Since the coefficient on the x^2 term is positive, the parabola opens upward, and the range will be all numbers greater than or equal to the *y*-value of the vertex, −6. The domain is all real numbers because any number can be substituted for *x* in *g*(*x*). The range is all real numbers because *g*(*x*) is a linear function and each *y*-value is used in a linear function.

B. **Correct expression for *f*(*g*(*x*)):** $4x^2 - 12x + 3$

Appropriate work leading to the answer:

$$f\left(g(x)\right) = f(2x-3) = (2x-3)^2 - 6 = 4x^2 - 12x + 9 - 6 = 4x^2 - 12x + 3$$

Explanation of the approach used to find the answer: I substituted in $2x - 3$ for *g*(*x*). Then, I substituted $2x - 3$ for each *x* in *g*(*x*). I squared $2x - 3$ using FOIL, then simplified and subtracted 6.

C. **Correct domain for *f*(*g*(*x*)):** All real numbers

Correct range for *f*(*g*(*x*)): $y \geq -6$

Appropriate work leading to the answer:

$$x = \frac{-(-12)}{2(4)} = \frac{12}{8} = \frac{3}{2}$$

$$f\left(g(x)\right) = f\left(g\left(\frac{3}{2}\right)\right) = 4\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 3 = 4\left(\frac{9}{4}\right) - 18 + 3 = 9 - 15 = -6$$

Explanation of the approach used to find the answer: The domain is all real numbers because any number can be substituted for *x* in *f*(*g*(*x*)). Also, the domain of *g*(*x*), the inner function, is all real numbers. The range is $y \geq -6$ because *f*(*g*(*x*)) is a parabola with its vertex at $\left(\frac{3}{2}, -6\right)$. Since the coefficient on the x^2 term is positive, the parabola opens upward, and the range will be all numbers greater than or equal to the *y*-value of the vertex, −6.

Rubric:

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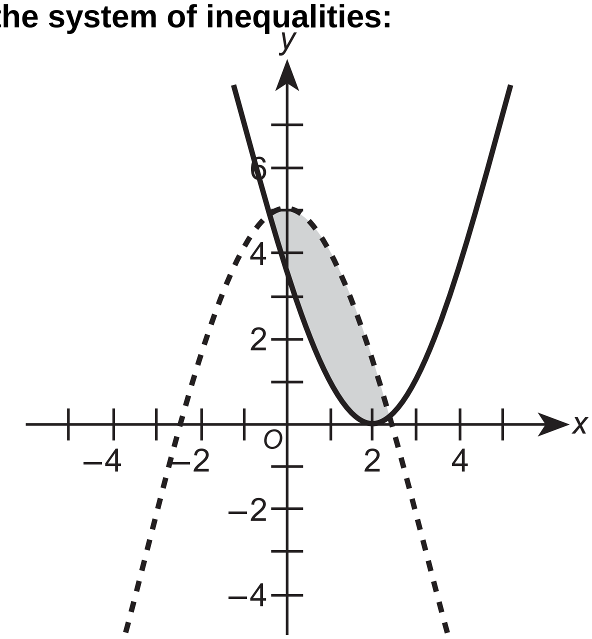
Scoring Criteria:

23)

A 4-point response may include, but is not limited to, the following points:

A. **Correct graph of the system of inequalities:**



Appropriate work needed to find the answer:

Points for the boundary equation of $y \geq (x-2)^2$:

$y = (-1-2)^2 = (-3)^2 = 9 \Rightarrow (-1,9)$

$y = (0-2)^2 = (-2)^2 = 4 \Rightarrow (0,4)$

$y = (1-2)^2 = (-1)^2 = 1 \Rightarrow (1,1)$

$y = (2-2)^2 = (0)^2 = 0 \Rightarrow (2,0)$

$y = (3-2)^2 = (1)^2 = 1 \Rightarrow (3,1)$

$y = (4-2)^2 = (2)^2 = 4 \Rightarrow (4,4)$

Note: Only 3 points are necessary and other points are possible.

Points for the boundary equation of $y < -x^2 + 5$:

$y = -(-2)^2 + 5 = -4 + 5 = 1 \Rightarrow (-2,1)$

$y = -(-1)^2 + 5 = -1 + 5 = 4 \Rightarrow (-1,4)$

$y = -(0)^2 + 5 = 0 + 5 = 5 \Rightarrow (0,5)$

$y = -(1)^2 + 5 = -1 + 5 = 4 \Rightarrow (1,4)$

$y = -(2)^2 + 5 = -4 + 5 = 1 \Rightarrow (2,1)$

$y = -(3)^2 + 5 = -9 + 5 = -4 \Rightarrow (3,-4)$

Note: Only 3 points are necessary and other points are possible.

Appropriate work needed to determine the direction of shading for the boundary equations:

Is $0 \geq (0-2)^2$?

Is $0 \geq (-2)^2$?

0 is not greater than or equal to 4

Is $0 < -(0)^2 + 5$?

0 is less than 5

B. **Correct points of intersection:** $\left(1-\frac{\sqrt{6}}{2}, \frac{5}{2}+\sqrt{6}\right)$ and $\left(1+\frac{\sqrt{6}}{2}, \frac{5}{2}-\sqrt{6}\right)$

Appropriate work needed to find the answer:

$(x-2)^2 = -x^2 + 5$

$x^2 - 4x + 4 = -x^2 + 5$

$2x^2 - 4x - 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)} = \frac{4 \pm \sqrt{16+8}}{4} = \frac{4 \pm \sqrt{24}}{4} = 1 \pm \frac{\sqrt{6}}{2}$$

$$y = \left(1-\frac{\sqrt{6}}{2}-2\right)^2 = \left(-1-\frac{\sqrt{6}}{2}\right)^2 = 1+2\left(\frac{\sqrt{6}}{2}\right)+\frac{6}{4} = \frac{5}{2}+\sqrt{6}$$

$$y = \left(1+\frac{\sqrt{6}}{2}-2\right)^2 = \left(-1+\frac{\sqrt{6}}{2}\right)^2 = 1-2\left(\frac{\sqrt{6}}{2}\right)+\frac{6}{4} = \frac{5}{2}-\sqrt{6}$$

Explanation of the approach used to find the answer: I set the boundary equations equal to each other. Then, I simplified the equation so that it was equal to 0. I used the quadratic formula to solve for *x*. Once I found the 2 values of *x*, I substituted them both back into $(x-2)^2$ and simplified. This gave me the 2 points of intersection.

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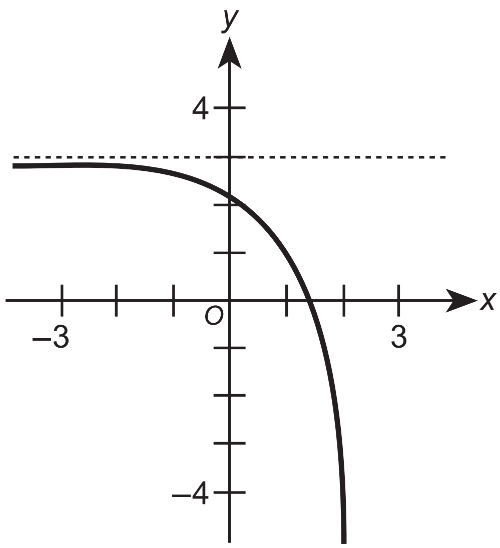
Scoring Criteria:

24)

A 4-point response may include, but is not limited to, the following points:

Correct graph:



Explanation of how the graph of the function was found using transformations: I started with the graph of $y = e^x$. Then, I shifted the graph 1 unit to the right because of $x - 1$. Then, I applied a vertical stretch by a factor of 2 because the absolute value of the coefficient of e^x is 2. Next, I reflected the graph across the *x*-axis because the coefficient of e^x is negative. Finally, I shifted the graph up 3 units because 3 is added to the transformed function (seen more clearly in the form $y = -2e^{x-1} + 3$).

Correct points on the graph: $\left(0, 3 - 2e^{-1}\right)$, $(1, 1)$, and $(2, 3 - 2e)$

Note: Other correct points are acceptable. Also, examinees may find points algebraically by applying the transformations of the graph to individual points from the original graph.

Appropriate work needed to find the correct points:

$x = 0 : y = 3 - 2e^{0-1} = 3 - 2e^{-1}$

$x = 1 : y = 3 - 2e^{1-1} = 3 - 2e^{0} = 3 - 2 = 1$

$x = 2 : y = 3 - 2e^{2-1} = 3 - 2e$

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