Name:	Date:
Teacher:	Class/Period:

### 1) What are the zeros of the quadratic function $f(x) = x^2 + 3x + 1$ ?

A. 
$$\frac{-3 \pm \sqrt{5}}{2}$$
  
B.  $\frac{-3 \pm \sqrt{13}}{2}$   
C.  $\frac{3 \pm \sqrt{5}}{2}$   
D.  $\frac{3 \pm \sqrt{13}}{2}$ 

- 2) The height above ground of an object thrown upward from an initial height of *s* ft with an initial velocity of *v* ft/sec is modeled by  $h(t) = -16t^2 + vt + s$ . Javier throws a baseball upward at 80 ft/sec from a platform 64 ft above the ground. To the nearest tenth of a second, when will the baseball hit the ground?
  - **A.** 0.7
  - **B.** 2.5
  - **C.** 5.0
  - **D.** 5.7
- 3) Monte has a small rectangular herb garden that has a length 3 yd longer than twice its width. If the area of the garden is 50 yd<sup>2</sup>, what is the length of the garden, to the nearest tenth of a yard?
  - **A.** 4.3
  - **B.** 7.3
  - **C.** 11.6
  - **D.** 37.4

4) What is the solution set to this inequality?

$$x^2 - 6x + 7 \le 2x - 5$$

**A.**  $\{x \mid x \le 2 \text{ or } x \ge 6\}$  **B.**  $\{x \mid x < 2 \text{ or } x > 6\}$  **C.**  $\{x \mid 2 \le x \le 6\}$ **D.**  $\{x \mid 2 < x < 6\}$ 

**5)** What are the solutions for  $x^2 = -4x + 7$ ?

- **A.**  $-2 \pm \sqrt{11}$  **B.**  $-4 \pm \sqrt{23}$  **C.** -7, 1 **D.**  $2 \pm \sqrt{3}$
- 6) The formula  $L = 0.1s^2 3s + 22$  gives the approximate runway length required to land a small plane. *L* is the length of the runway, in feet, and *s* is the landing speed of the airplane, in feet per second. The pilot knows that the runway is 2,400 ft long. To the nearest foot per second, what is the maximum safe landing speed?
  - **A**. 50
  - **B.** 90
  - **C.** 140
  - **D.** 170
- 7) Given  $(k 1)x^2 + kx + 1 = 0$ , where -1 is one solution, what is the other solution?
  - **A.** 1

**B.** 
$$-\frac{1}{3}$$
  
**C.**  $\frac{-1}{k-1}$ 

**D.** 
$$\frac{-k(k-2)}{2(k-1)}$$

- 8) For what values of c will  $x^2 + 4x + c = 0$  have 2 complex conjugate roots?
  - **A.** *c* < 2
  - **B.** c > 2
  - **C.** *c* < 4
  - **D.** c > 4
- 9) For the equation  $x^2 4x + 4 = 9$ , determine the discriminant.
  - **A.** -36
  - **B.** 0
  - **C.** 6
  - **D.** 36
- **10)** Which quadratic equation has only nonreal complex roots?
  - **A.**  $x^2 7x 12 = 0$  **B.**  $x^2 + 8x + 10 = 0$  **C.**  $x^2 - x + 5 = 0$ **D.**  $x^2 + 4x - 1 = 0$
- **11)** One zero of  $f(x) = x^4 + x^3 2x^2 + 4x 24$  is 2*i*. What are the other zeros of this function?
  - A. -2*i*, -3, 2
    B. -2*i*, 3, -2
    C. -4*i*, -3, 2
    D. -4*i*, 3, -2
- **12)** What is the solution set for  $5t^2 + 6 = 8t$ ?

**A.** 
$$\{-\frac{3}{5} \pm \frac{i}{5}\sqrt{31}\}$$
  
**B.**  $\{\frac{4}{5} \pm \frac{2}{5}i\sqrt{14}\}$   
**C.**  $\{-\frac{4}{5} \pm \frac{i}{5}\sqrt{14}\}$   
**D.**  $\{\frac{4}{5} \pm \frac{i}{5}\sqrt{14}\}$ 

13) What are the roots of this equation?

$$x^2 + 2x + 12 = 0$$

**A.** 
$$-1 \pm i\sqrt{11}$$
  
**B.**  $-2 \pm i\sqrt{11}$   
**C.**  $-2 \pm 2i\sqrt{11}$   
**D.**  $-1 \pm 2i\sqrt{11}$ 

**14)** What is the solution set of  $2 v^2 + 1 = x - \frac{1}{2}$ ?

$$\frac{2}{3}x^{2} + 1 = x - \frac{1}{2}x^{2}$$
A.  $\{\frac{3\pm 3i\sqrt{3}}{4}\}$ 
B.  $\{\frac{3\pm 3\sqrt{3}}{4}\}$ 
C.  $\{\frac{-3\pm 3i\sqrt{3}}{4}\}$ 
D.  $\{\frac{-3\pm i\sqrt{3}}{4}\}$ 

15) This system of equations:

$$\begin{cases} y-3 = -(x+5)(1-x) \\ 2x^2 - y + 3 = 0 \end{cases}$$

has no real-valued solutions; if the equations were graphed in the x-y plane, they would be represented by two parabolas that do not intersect. But in the complex numbers, there are two solutions.

What are the values of *x* for the solution set to this system?

- **A.** {-1, 5}
- **B.** {± *i*}
- **C.**  $\{-2 \pm i\}$
- **D.** {2 ± *i*}

- **16)** What condition will yield non-real zeros of a quadratic function  $f(x) = ax^2 + bx + c$ ?
  - **A.**  $-b < b^2$  **B.**  $b^2 < 4ac$ **C.** 2a < 0
  - **D.**  $b^2 > 4ac$

**17)** What are the solutions of  $4x^2 = 3x - 2$ ?

A. 
$$\frac{-3 \pm \sqrt{41}}{8}$$
  
B.  $\frac{3 \pm 4\sqrt{2}}{8}$   
C.  $\frac{3 \pm i\sqrt{35}}{8}$   
D.  $\frac{3 \pm i\sqrt{23}}{8}$ 

- **18)** Find all solutions to the equation  $-2x^3 3x^2 + 5x + 6 = 0$ , rounding decimals to the nearest 0.1.
  - **A.** -3.0, 2.0

- **C.** -2.0, 3.0
- **D.** (-0.5 + 2.1*i*), (-0.5 2.1*i*), 2.8
- **19)** What are the 2 points of intersection for this system of equations?

$$\begin{cases} x^2 + y^2 = 25\\ (x - 4)^2 + y^2 = 9 \end{cases}$$

- **A.** (4,3), (4,-3)
- **B.** (0,5), (4,3)
- **C.** (0,-5), (4,-3)
- **D.** (5,3), (-5,-3)

**20)** Given x > 0, at which value of x will  $y_2 - y_1 = 2$ ?

$$y_1 = 2x^2 - 2x + 5$$
  

$$y_2 = 3x^2 - 5x + 7$$
  
A. 2  
B. 3  
C. 4

- **D.** 5
- **21)** What is the total number of solutions for this system of equations?

$$\begin{cases} (x-2)^2 + (y+3)^2 = 4\\ (y+6) = -(x-2)^2 \end{cases}$$

- **A.** 0
- **B.** 1
- **C.** 2
- **D.** 4

- **22)** Ethan is working with the quadratic function  $y = x^2 5x + 10$ .
  - A. Ethan needs to determine the number and type of roots for the equation using the discriminant. His solution is shown. Explain what Ethan did correctly and what he did incorrectly, and then correct his mistakes.

1.	$b^2 - 4ac = -5^2 - 4(1)(10) = -25 - 40 = -65$
2.	Since $-65 < 0$ , there are 2 nonreal solutions.

B. Ethan decides to complete the square to rewrite the quadratic equation in vertex form and find the solutions to the equation when y = 0. His solution is shown. Identify what Ethan did incorrectly, and then explain what he should have done instead. (Note: identify each error once)

1. 
$$y - 10 = x^2 - 5x + 10 - 10 \rightarrow y - 10 = x^2 - 5x$$
  
2.  $-5^2 = -25$   
3.  $y - 10 - 25 = x^2 - 5x - 25$   
4.  $y - 35 = (x - 5)^2$   
5.  $y = (x - 5)^2 - 35$   
6.  $0 = (x - 5)^2 - 35$   
7.  $(x - 5)^2 = 35$   
8.  $x - 5 = \frac{35}{2}$  or  $x - 5 = -\frac{35}{2}$   
9.  $x = \frac{35}{2} + \frac{10}{2}$  or  $x = -\frac{35}{2} + \frac{10}{2}$   
10.  $x = \frac{45}{2}$  or  $x = -\frac{25}{2}$ 

C. Do the solutions Ethan found in Part B match the answer he gave in Part A? Explain why or why not.

23) Given the system of inequalities shown:

$$\begin{cases} y \ge (x-2)^2 \\ y < -x^2 + 5 \end{cases}$$

- A. Graph the system of inequalities. Show the work you used to obtain at least 3 points on the boundary equation of each graph and to decide the direction of the shading for each boundary equation.
- B. Find the exact points of intersection for each boundary equation in the system of inequalities. Show your work algebraically, and explain the approach you used to find your answer.
- **24)** Consider the graph of the function  $f(x) = x^4 25x^2 36x$ , which has one *x*-intercept at (-4,0). Find all the other zeros of the function algebraically. Show your work, and explain the approach you used to find your answer.



**25)** The general form of a particular circle is  $x^2 + y^2 - 4x + 2y - 11 = 0$ .

- A. What is the standard form of the equation of the circle? Show your work algebraically, and explain the approach you used to find your answer.
- B. Determine the coordinates for the center of the circle. Explain how you determined your answer.
- C. Determine the *x*-coordinates of each point on the circle with a *y*-coordinate 2 less than the center's *y*-coordinate. Simplify your answer completely. Show your algebraic work, and explain the approach you used to find your answer.

Please use the space below to write your response(s) to the writing assignment provided by your teacher. If there are multiple tasks to the question, please clearly label the number or letter of each task in the column to the left of your answers. If you need additional pages for your response, your teacher can provide them.

Please write the name of the writing assignment here: \_\_\_\_\_

#### Task

### **QualityCore**<sup>®</sup>

# Reference Sheet for the QualityCore<sup>™</sup> Algebra II End-of-Course Assessment

Equations of a Line			
Standard Form	Ax + By = C	A, B, and C are constants with A and B not	
Slope-Intercept Form	$y = mx + b$ $y - y_1 = m(x - x_1)$	both equal to zero. (x <sub>1</sub> ,y <sub>1</sub> ) is a point. <i>m</i> = slope <i>b</i> = <i>y</i> -intercept	
Point-Slope Form			
Quadratics			
Standard Form of a Quadratic Equation	$ax^2 + bx + c = 0$	<i>a</i> , <i>b</i> , and <i>c</i> are constants, where $a \neq 0$ .	
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		
Conic Sections			
Circle	$(x - h)^2 + (y - k)^2 = r^2$	center ( <i>h,k</i> ) <i>r</i> = radius	
		axis of symmetry $x = h$ vertex $(h,k)$	
Parabola	$y = a(x-h)^2 + k$	directrix $y = k - \frac{1}{4a}$ focus $\left(h, k + \frac{1}{4a}\right)$	
		axis of symmetry $y = k$ vertex $(h,k)$	
Parabola	$x = a(y-k)^2 + h$	directrix $x = h - \frac{1}{4a}$ focus $(h + \frac{1}{4a}, k)$	
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 - b^2$ , center $(h,k)$	
Ellipse	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$	foci (h, $k \pm c$ ) where $c^2 = a^2 - b^2$ , center (h,k)	
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	foci $(h \pm c, k)$ where $c^2 = a^2 + b^2$ , center $(h,k)$	
Hyperbola	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	foci (h, $k \pm c$ ) where $c^2 = a^2 + b^2$ , center (h,k)	
Lines and Points			
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	$(x_1, y_1)$ and $(x_2, y_2)$ are 2 points.	
Midpoint	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	m = slope M = midpoint d = distance	

Distance

 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 

#### Miscellaneous

Distance, Rate, Time	D = rt	D = distance
Simple Interest Compound Interest	$I = prt$ $A = p\left(1 + \frac{r}{n}\right)^{nt}$	r = rate t = time l = interest p = principal A = amount of money after t years n = number of times interest is compounded annually
Pythagorean Theorem	$a^2 + b^2 = c^2$	a and b = legs of right triangle c = hypotenuse

#### Laws of Sines and Cosines

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc\cos A$



#### Sequences, Series, and Counting

Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Arithmetic Series	$s_n = \frac{n}{2}(a_1 + a_n)$
Geometric Sequence	$a_n = a_1(r^{n-1})$
Geometric Series	$s_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Combinations	$_{k}C_{m} = C(k,m) = \frac{k!}{(k-m)! \ m!}$
Permutations	$_{k}P_{m}=P(k,m)=\frac{k!}{(k-m)!}$

$a_n = n^{\text{th}} \text{ term}$
n = number of the term
d = common difference
$s_n = $ sum of the first <i>n</i> terms
<i>r</i> = common ratio
k = number of objects in the set
m = number of objects selected

#### Circumference, Area, and Volume

Triangle	$A = \frac{1}{2}bh$	A = area
Parallelogram	A = bh	$b = base$ $h = height$ $r = radius$ $C = circumference$ $d = diameter$ $V = volume$ $B = area of base$ $\pi \approx 3.14$
Trapezoid	$A=\frac{1}{2}(b_1+b_2)h$	
Circle	$A = \pi r^2$ $C = \pi d$	
General Prism	V = Bh	
Right Circular Cylinder	$V = \pi r^2 h$	
Pyramid	$V = \frac{1}{3}Bh$	
Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$	
Sphere	$V = \frac{4}{3}\pi r^3$	



- 1) A
- 2) D
- 3) C
- 4) C
- 5) A 6) D
- 7) C
- 8) D
- 9) D
- 10) C
- 11) A
- 12) D
- 13) A
- 14) A
- 15) D
- 16) B
- 17) D
- 18) B
- 19) A
- 20) B
- 21) A

### 22)

A 4-point response may include, but is not limited to, the following points:

**A**. **Explanation of what Ethan did correctly:** In step 2, Ethan said that there are 2 nonreal solutions because the value he computed for the discriminant is less than 0.

**Explanation of what Ethan did incorrectly:** In step 1, Ethan only squared the 5 and left the negative sign in front of the 5. He should have calculated  $(-5)^2 - 4(1)(10) = 25 - 40 = -15$ .

# B. Identification of what Ethan did incorrectly, and explanation of what he should have done: In step 2, Ethan attempted to square -5. He should have squared $-\frac{5}{2}$

instead of -5. His result should have been  $\left(-\frac{5}{2}\right)^2 = \frac{25}{4}$ . Also in step 2, Ethan only

squared 5. He forgot to square -5, which would have given him 25. In step 4, Ethan factored  $x^2 - 5x - 25$  incorrectly. The expression  $x^2 - 5x - 25$  is not factorable. In step 5, Ethan added 35 to the left-hand side of the equation but subtracted 35 from the right-hand side of the equation. He should have added 35 to both sides of the equation. In step 8, Ethan divided 35 by 2. He should have found the square root of 35.

### C. Determination of whether Ethan's solutions match his answer to part A: No

**Explanation of why the determination was made:** Ethan's solutions do not match his answer to part A because  $\frac{45}{2}$  and  $-\frac{25}{2}$  are not nonreal solutions.

# 4 A response at this level provides evidence of thorough knowledge and understanding of the subject matter.

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.
- 3 A response at this level provides evidence of competent knowledge and understanding of the subject matter.
  - The response addresses most parts of the question or problem correctly.
  - The response includes some minor errors but generally uses appropriate procedures accurately.
  - The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
  - The explanation in the response is mostly clear and coherent.

# 2 A response at this level provides evidence of a basic knowledge and understanding of the subject matter.

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.
- **0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

23)

A 4-point response may include, but is not limited to, the following points:

A. Correct graph of the system of inequalities:



### Appropriate work needed to find the answer:

Points for the boundary equation of 
$$y \ge (x-2)^2$$
:  
 $y = (-1-2)^2 = (-3)^2 = 9 \Rightarrow (-1,9)$   
 $y = (0-2)^2 = (-2)^2 = 4 \Rightarrow (0,4)$   
 $y = (1-2)^2 = (-1)^2 = 1 \Rightarrow (1,1)$   
 $y = (2-2)^2 = (0)^2 = 0 \Rightarrow (2,0)$   
 $y = (3-2)^2 = (1)^2 = 1 \Rightarrow (3,1)$   
 $y = (4-2)^2 = (2)^2 = 4 \Rightarrow (4,4)$ 

Note: Only 3 points are necessary and other points are possible.

Points for the boundary equation of  $y < -x^2 + 5$ :  $y = -(-2)^2 + 5 = -4 + 5 = 1 \Rightarrow (-2,1)$   $y = -(-1)^2 + 5 = -1 + 5 = 4 \Rightarrow (-1,4)$   $y = -(0)^2 + 5 = 0 + 5 = 5 \Rightarrow (0,5)$   $y = -(1)^2 + 5 = -1 + 5 = 4 \Rightarrow (1,4)$   $y = -(2)^2 + 5 = -4 + 5 = 1 \Rightarrow (2,1)$  $y = -(3)^2 + 5 = -9 + 5 = -4 \Rightarrow (3,-4)$ 

Note: Only 3 points are necessary and other points are possible.

### Appropriate work needed to determine the direction of shading for the boundary equations:

Is  $0 \ge (0-2)^2$ ? Is  $0 \ge (-2)^2$ ? 0 is not greater than or equal to 4 Is  $0 < -(0)^2 + 5$ ? 0 is less than 5

**B**. Correct points of intersection:  $\left(1 - \frac{\sqrt{6}}{2}, \frac{5}{2} + \sqrt{6}\right)$  and  $\left(1 + \frac{\sqrt{6}}{2}, \frac{5}{2} - \sqrt{6}\right)$ 

Appropriate work needed to find the answer:

$$(x-2)^{2} = -x^{2} + 5$$

$$x^{2} - 4x + 4 = -x^{2} + 5$$

$$2x^{2} - 4x - 1 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^{2} - 4(2)(-1)}}{2(2)} = \frac{4 \pm \sqrt{16 + 8}}{4} = \frac{4 \pm \sqrt{24}}{4} = 1 \pm \frac{\sqrt{6}}{2}$$

$$y = \left(1 - \frac{\sqrt{6}}{2} - 2\right)^{2} = \left(-1 - \frac{\sqrt{6}}{2}\right)^{2} = 1 + 2\left(\frac{\sqrt{6}}{2}\right) + \frac{6}{4} = \frac{5}{2} + \sqrt{6}$$

$$y = \left(1 + \frac{\sqrt{6}}{2} - 2\right)^{2} = \left(-1 + \frac{\sqrt{6}}{2}\right)^{2} = 1 - 2\left(\frac{\sqrt{6}}{2}\right) + \frac{6}{4} = \frac{5}{2} - \sqrt{6}$$

**Explanation of the approach used to find the answer:** I set the boundary equations equal to each other. Then, I simplified the equation so that it was equal to 0. I used the quadratic formula to solve for *x*. Once I found the 2 values of *x*, I substituted them both back into  $(x-2)^2$  and simplified. This gave me the 2 points of intersection.

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- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.
- 3 A response at this level provides evidence of competent knowledge and understanding of the subject matter.
  - The response addresses most parts of the question or problem correctly.
  - The response includes some minor errors but generally uses appropriate procedures accurately.
  - The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
  - The explanation in the response is mostly clear and coherent.

# 2 A response at this level provides evidence of a basic knowledge and understanding of the subject matter.

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.
- **0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.

24)

A 4-point response may include, but is not limited to, the following points:

Correct additional zeros: 0,  $2\pm\sqrt{13}$ 

#### Appropriate work needed to find the answer:

 $f(x) = x^{4} - 25x^{2} - 36x = x(x^{3} - 25x - 36)$   $\underline{-4} \quad 1 \quad 0 \quad -25 \quad -36$   $\underline{-4 \quad 16 \quad 36}$   $1 \quad -4 \quad -9 \quad \underline{0}$   $x^{2} - 4x - 9 = 0$   $x = \frac{-(-4) \pm \sqrt{(-4)^{2} - 4(1)(-9)}}{2(1)} = \frac{4 \pm \sqrt{16 + 36}}{2} = \frac{4 \pm \sqrt{52}}{2} = \frac{4 \pm 2\sqrt{13}}{2} = 2 \pm \sqrt{13}$ 

**Note:** An examinee could also use polynomial long division instead of synthetic division to find the answer.

**Explanation of the approach used to find the answer:** First, I factored out an *x*. That gave me x = 0 as a zero of the function. Since (-4,0) is an *x*-intercept of the graph of the function, x = -4 is a zero of the function. Since x = -4 is a zero of the function, I used synthetic division to find  $\frac{x^3 - 25x - 36}{x + 4}$ . I put in 0 for the coefficient of the  $x^2$  term in the synthetic division process. Performing synthetic division told me that  $\frac{x^3 - 25x - 36}{x + 4} = x^2 - 4x - 9$ . Then, I used

the quadratic formula to find the remaining zeros.

# 4 A response at this level provides evidence of thorough knowledge and understanding of the subject matter.

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- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
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- Many parts of the explanation are difficult to understand.
- **0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.



A 4-point response may include, but is not limited to, the following points:

### A. Correct equation: $(x-2)^2 + (y+1)^2 = 16$

#### Appropriate work needed to find the answer:

$$x^{2} - 4x + y^{2} + 2y - 11 = 0$$
  

$$x^{2} - 4x + y^{2} + 2y = 11$$
  

$$x : \left(\frac{-4}{2}\right)^{2} = (-2)^{2} = 4$$
  

$$y : \left(\frac{2}{2}\right)^{2} = (1)^{2} = 1$$
  

$$x^{2} - 4x + 4 + y^{2} + 2y + 1 = 11 + 4 + 1$$
  

$$(x - 2)^{2} + (y + 1)^{2} = 16$$

**Explanation of the approach used to find the answer:** I collected the *x* terms and the *y* terms. I added 11 to both sides of the equation, and completed the square for *x* and *y*. I used the following steps: I took half of the coefficient on the linear term. I squared those numbers and added both of them to both sides of the equation. I wrote the expression on the left-hand side as 2 perfect square trinomials.

### **B**. **Correct center:** (2,-1)

**Explanation of how the answer was determined:** I took the expressions inside each set of parentheses and solved for the variable to find each coordinate ( $x - 2 = 0 \Rightarrow x = 2$  and  $y + 1 = 0 \Rightarrow y = -1$ ).

#### OR

The standard form of an equation of a circle is  $(x-h)^2 + (y-k)^2 = r^2$ , where (h,k) is the center.

### **C.** Correct *x*-coordinates: $x = 2 \pm 2\sqrt{3}$

#### Appropriate work needed to find the answer:

-1-2 = -3  $(x-2)^{2} + (-3+1)^{2} = 16$   $(x-2)^{2} + (-2)^{2} = 16$   $(x-2)^{2} + 4 = 16$   $(x-2)^{2} = 12$   $x-2 = \pm\sqrt{12}$  $x-2 = \pm\sqrt{3}$ 

**Explanation of the approach used to find the answer:** I subtracted 2 from -1 to get -3 to get the new *y*-coordinate. I substituted -3 for *y* in the standard-form equation. I

simplified the expression. I took the square root of both sides of the equation, simplified the radical, and added 2 to both sides of the equation.

# 4 A response at this level provides evidence of thorough knowledge and understanding of the subject matter.

- The response addresses all parts of the question or problem correctly.
- The response demonstrates efficient and accurate use of appropriate procedures.
- The explanation of strategies used in the response shows evidence of a good understanding of mathematical concepts and principles, and it does not contain any misconceptions.
- The explanation in the response is clear and coherent.
- 3 A response at this level provides evidence of competent knowledge and understanding of the subject matter.
  - The response addresses most parts of the question or problem correctly.
  - The response includes some minor errors but generally uses appropriate procedures accurately.
  - The explanation of strategies used in the response shows some evidence of a good understanding of mathematical concepts and principles, and it contains few, if any, misconceptions.
  - The explanation in the response is mostly clear and coherent.

# 2 A response at this level provides evidence of a basic knowledge and understanding of the subject matter.

- The response addresses some parts of the question or problem correctly.
- The response includes a number of errors but demonstrates some use of appropriate procedures.
- The explanation of strategies used in the response shows a little evidence of understanding of mathematical concepts and principles, but it may contain some evidence of misconceptions.
- The explanation in the response is partially clear, but some parts may be difficult to understand.

- The response addresses a few parts of the problem correctly, but the response is mostly incorrect.
- The response includes inappropriate procedures or simple manipulations that show little or no understanding of correct procedures.
- The explanation of strategies used in the response shows little or no evidence of understanding of mathematical concepts and principles, and it may contain evidence of significant misconceptions.
- Many parts of the explanation are difficult to understand.
- **0 A response at this level is not scorable.** The response is off-topic, blank, hostile, or otherwise not scorable.