Key

Chapter Summary

In this chapter, we have reviewed basic skills for finding indefinite integrals. We've looked at the antiderivative formulas for all of the basic functions and reviewed techniques for finding antiderivatives of other functions.

We've also reviewed the more advanced techniques of integration by partial fractions and integration by parts, both topics only for the BC Calculus course .

Practice Exercises

Directions: Answer these questions *without* using your calculator.

#1 $\frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}} + 3x + C$

1. $\int (3x^2 - 2x + 3)\,dx =$

(A) $x^3 - x^2 + C$ (B) $3x^3 - x^2 + 3x + C$ (C) $x^3 - x^2 + 3x + C$

(D) $\frac{1}{2}(3x^2 - 2x + 3)^2 + C$ (E) none of these

#2 $= \int \left(x^2 - 1 + \frac{1}{4x^2}\right) dx$

Expand

2. $\int \left(x - \frac{1}{2x}\right)^2 dx =$ $\left(x - \frac{1}{2x}\right)\left(x - \frac{1}{2x}\right) = x^2 - \frac{1}{2x}(x) - \frac{1}{2x}(x) + \frac{1}{4x^2}$

$= \int \left(x^2 - 1 + \frac{1}{4}x^{-2}\right)$

$= \frac{x^3}{3} - x + \frac{1}{4}\left(\frac{x^{-1}}{-1}\right) + C$

(A) $\frac{1}{3}\left(x - \frac{1}{2x}\right)^3 + C$ (B) $x^2 - 1 + \frac{1}{4x^2} + C$ (C) $\frac{x^3}{3} - 2x - \frac{1}{4x} + C$

(D) $\frac{x^3}{3} - x - \frac{4}{x} + C$ (E) none of these

$= \frac{x^3}{3} - x - \frac{1}{4x} + C$

#3

3. $-\frac{1}{2}\int \sqrt{4 - 2t}\,{-2}\,dt =$ $u = 4 - 2t$, $du = -2\,dt$

$-\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{\cancel{2}}\,\frac{\cancel{2}u^{3/2}}{3} + C$

$= -\frac{(4-2t)^{3/2}}{3} + C$

(A) $-\frac{1}{3}(4 - 2t)^{3/2} + C$ (B) $\frac{2}{3}(4 - 2t)^{3/2} + C$ (C) $-\frac{1}{6}(4 - 2t)^3 + C$

(D) $+\frac{1}{2}(4 - 2t)^2 + C$ (E) $\frac{4}{3}(4 - 2t)^{3/2} + C$

4. $\int (2 - 3x)^5\,dx =$

(A) $\frac{1}{6}(2 - 3x)^6 + C$ (B) $-\frac{1}{2}(2 - 3x)^6 + C$ (C) $\frac{1}{2}(2 - 3x)^6 + C$

(D) $-\frac{1}{18}(2 - 3x)^6 + C$ (E) none of these

#5

5. $\frac{1}{2}\int \frac{1 - 3y}{\sqrt{2y - 3y^2}}\,2\,dy =$ $u = 2y - 3y^2$, $du = (2 - 6y)\,dy = 2(1 - 3y)\,dy$

$\frac{1}{2}\int \frac{1}{u^{1/2}}\,du = \frac{1}{2}\int u^{-1/2}\,du$

$= \frac{1}{\cancel{2}}\,\frac{\cancel{2}u^{1/2}}{1}$

(A) $4\sqrt{2y - 3y^2} + C$ (B) $\frac{1}{4}(2y - 3y^2)^2 + C$ (C) $\frac{1}{2}\ln\sqrt{2y - 3y^2} + C$

(D) $\frac{1}{4}(2y - 3y^2)^{1/2} + C$ (E) $\sqrt{2y - 3y^2} + C$

$= \sqrt{2y - 3y^2} + C$

6. $\int \frac{dx}{3(2x-1)^2} =$

(A) $\frac{-3}{2x-1} + C$ (B) $\frac{1}{6-12x} + C$ (C) $+\frac{6}{2x-1} + C$

(D) $\frac{2}{3\sqrt{2x-1}} + C$ (E) $\frac{1}{3}\ln|2x-1| + C$

#7 $u = 1+3x$
$du = 3dx$

$\frac{2}{3}\int \frac{1}{u}\,du$

$\frac{2}{3}\ln u + C$

7. $\int \frac{2\,d\cancel{u}x}{1+3\cancel{u}x} =$ $2\int \frac{dx}{1+3x}$

(A) $\frac{2}{3}\ln|1+3u| + C$ (B) $-\frac{1}{3(1+3u)^2} + C$ (C) $2\ln|1+3u| + C$

(D) $\frac{3}{(1+3u)^2} + C$ (E) none of these

8. $\int \frac{t}{\sqrt{2t^2-1}}\,dt =$

(A) $\frac{1}{2}\ln\sqrt{2t^2-1} + C$ (B) $4\ln\sqrt{2t^2-1} + C$ (C) $8\sqrt{2t^2-1} + C$

(D) $-\frac{1}{4(2t^2-1)} + C$ (E) $\frac{1}{2}\sqrt{2t^2-1} + C$

#9 $u = 3x$
$du = 3dx$

$\frac{1}{3}\int \cos u\,du$

$\frac{1}{3}\sin(3x) + C$

9. $\frac{1}{3}\int \cos 3x\,dx =$

(A) $3\sin 3x + C$ (B) $-\sin 3x + C$ (C) $-\frac{1}{3}\sin 3x + C$

(D) $\frac{1}{3}\sin 3x + C$ (E) $\frac{1}{2}\cos^2 3x + C$

10. $\int \frac{x\,dx}{1+4x^2} =$

(A) $\frac{1}{8}\ln(1+4x^2) + C$ (B) $\frac{1}{8(1+4x^2)^2} + C$ (C) $\frac{1}{4}\sqrt{1+4x^2} + C$

(D) $\frac{1}{2}\ln|1+4x^2| + C$ (E) $\frac{1}{2}\tan^{-1}2x + C$

#11 $u^2 = 4x^2$
$u = 2x$
$du = 2dx$

$\frac{1}{2}\int \frac{du}{1+u^2}$

$\frac{1}{2}\tan^{-1}(u) + C$

$\frac{1}{2}\tan^{-1}(2x) + C$

11. $\frac{1}{2}\int \frac{dx}{1+4x^2} =$ make look like $\int \frac{du}{1+u^2}$

(A) $\tan^{-1}(2x) + C$ (B) $\frac{1}{8}\ln(1+4x^2) + C$ (C) $\frac{1}{8(1+4x^2)^2} + C$

(D) $\frac{1}{2}\tan^{-1}(2x) + C$ (E) $\frac{1}{8x}\ln|1+4x^2| + C$

12. $\frac{1}{8}\int \frac{8x}{(1+4x^2)^2}\,dx =$

$u = 1+4x^2$

$du = 8x\,dx$

$\frac{1}{8}\int \frac{du}{u^2} = \frac{1}{8}\int u^{-2}\,du$

$= \frac{1}{8}\,\frac{u^{-1}}{-1} + C$

$= -\frac{1}{8u} + C$

(A) $\frac{1}{8}\ln(1+4x^2)^2 + C$ (B) $\frac{1}{4}\sqrt{1+4x^2} + C$ (C) $-\frac{1}{8(1+4x^2)} + C$

(D) $-\frac{1}{3(1+4x^2)^3} + C$ (E) $-\frac{1}{(1+4x^2)} + C$

13. $\int \frac{x\,dx}{\sqrt{1+4x^2}} =$

(A) $\frac{1}{8}\sqrt{1+4x^2} + C$ (B) $\frac{\sqrt{1+4x^2}}{4} + C$ (C) $\frac{1}{2}\sin^{-1} 2x + C$

(D) $\frac{1}{2}\tan^{-1} 2x + C$ (E) $\frac{1}{8}\ln\sqrt{1+4x^2} + C$

14. $\int \frac{dy}{\sqrt{4-y^2}} =$ need in form $\frac{1}{\sqrt{1+u^2}}$ So $2\int \frac{\frac{1}{2}\,dy}{\sqrt{4\left(1-\left(\frac{y}{2}\right)^2\right)}}$

$u = \frac{y}{2}$

$du = \frac{1}{2}\,dy$

$2\int \frac{du}{2\sqrt{1+u^2}}$

$\sin^{-1} u + C$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2} + C$ (B) $-\sqrt{4-y^2} + C$ (C) $\sin^{-1}\frac{y}{2} + C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2} + C$ (E) $-\frac{1}{3(4-y^2)^{3/2}} + C$

15. $\int \frac{y\,dy}{\sqrt{4-y^2}} =$

(A) $\frac{1}{2}\sin^{-1}\frac{y}{2} + C$ (B) $-\sqrt{4-y^2} + C$ (C) $\sin^{-1}\frac{y}{2} + C$

(D) $-\frac{1}{2}\ln\sqrt{4-y^2} + C$ (E) $2\sqrt{4-y^2} + C$

16. $\int \frac{2x+1}{2x}\,dx =$ $\int \frac{2x}{2x}\,dx + \int \frac{1}{2x}\,dx = \int dx + \frac{1}{2}\int \frac{1}{x}\,dx$

(A) $x + \frac{1}{2}\ln|x| + C$ (B) $1 + \frac{1}{2}x^{-1} + C$ (C) $x + 2\ln|x| + C$

$x + \frac{1}{2}\ln|x| + C$

(D) $x + \ln|2x| + C$ (E) $\frac{1}{2}\left(2x - \frac{1}{x^2}\right) + C$

17. $\int \frac{(x-2)^3}{x^2}\,dx =$ Expand $\frac{(x-2)(x-2)(x-2)}{x^2} \rightarrow \frac{x-2(x^2-4x+4)}{x^2} \rightarrow \frac{x^3-4x^2+4x-2x^2+8x-8}{x^2}$

(A) $\frac{(x-2)^4}{4x^2} + C$ (B) $\frac{x^2}{2} - 6x + \cancel{6}\,12\ln|x| - \frac{8}{x} + C$

$\int \frac{x^3-6x^2+12x-8}{x^2}\,dx$

(C) $\frac{x^2}{2} - 3x + 6\ln|x| + \frac{4}{x} + C$ (D) $-\frac{(x-2)^4}{4x} + C$

(E) none of these

$\int \frac{x^3}{x^2} - \int \frac{6x^2}{x^2} + \int \frac{12x}{x^2} - \int \frac{8}{x^2} = \int x - \int 6 + 12\int \frac{1}{x} - \int 8x^{-2}$

$\frac{x^2}{2} - 6x + 12\ln|x| - \frac{8x^{-1}}{-1}$

18. $\int \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 dt =$

(A) $t - 2 + \frac{1}{t} + C$ **(B)** $\frac{t^3}{3} - 2t - \frac{1}{t} + C$ **(C)** $\frac{t^2}{2} + \ln|t| + C$

(D) $\frac{t^2}{2} - 2t + \ln|t| + C$ **(E)** $\frac{t^2}{2} - t - \frac{1}{t^2} + C$

19. $\int (4x^{1/3} - 5x^{3/2} - x^{-1/2})\, dx =$ $4\frac{3x^{4/3}}{4} - 5\frac{2x^{5/2}}{5} - \frac{2x^{1/2}}{1} + C$

(A) $3x^{4/3} - 2x^{5/2} - 2x^{1/2} + C$
(B) $3x^{4/3} - 2x^{5/2} + 2x^{1/2} + C$
(C) $6x^{2/3} - 2x^{5/2} - \frac{1}{2}x^2 + C$
(D) $\frac{4}{3}x^{-2/3} - \frac{15}{2}x^{1/2} + \frac{1}{2}x^{-3/2} + C$
(E) none of these

20. $\int \frac{x^3 - x - 1}{x^2}\, dx =$ $\int \frac{x^{\cancel{3}}}{\cancel{x^2}} - \int \frac{\cancel{x}}{x^2} - \int \frac{1}{x^2}$

$\int x\,dx - \int \frac{1}{x}dx - \int x^{-2}dx$

$\frac{x^2}{2} - \ln|x| - \frac{x^{-1}}{-1} + C$

$\frac{x^2}{2} - \ln|x| + \frac{1}{x} + C$

(A) $\dfrac{\frac{1}{4}x^4 - \frac{1}{2}x^2 - x}{\frac{1}{3}x^3} + C$

(B) $1 + \frac{1}{x^2} + \frac{2}{x^3} + C$

(C) $\frac{x^2}{2} - \ln|x| - \frac{1}{x} + C$

(D) $\frac{x^2}{2} - \ln|x| + \frac{1}{x} + C$

(E) $\frac{x^2}{2} - \ln|x| + \frac{2}{x^3} + C$

21. $\int \frac{dy}{\sqrt{y}(1 - \sqrt{y})} =$

(A) $4\sqrt{1 - \sqrt{y}} + C$ **(B)** $\frac{1}{2}\ln\left|1 - \sqrt{y}\right| + C$ **(C)** $2\ln(1 - \sqrt{y}) + C$

(D) $2\sqrt{y} - \ln|y| + C$ **(E)** $-2\ln\left|1 - \sqrt{y}\right| + C$

22. $\int \frac{u\, du}{\sqrt{4 - 9u^2}} =$

(A) $\frac{1}{3}\sin^{-1}\frac{3u}{2} + C$ **(B)** $-\frac{1}{18}\ln\sqrt{4 - 9u^2} + C$ **(C)** $2\sqrt{4 - 9u^2} + C$

(D) $\frac{1}{6}\sin^{-1}\frac{3}{2}u + C$ **(E)** $-\frac{1}{9}\sqrt{4 - 9u^2} + C$

$u = \sin\theta$
$du = \cos\theta\, d\theta$ $\qquad \int u\, du$

23. $\int \sin\theta \cos\theta \, d\theta =$ $\qquad = \frac{u^2}{2} + C$

(A) $-\frac{\sin^2\theta}{2} + C$ **(B)** $-\frac{1}{4}\cos 2\theta + C$ **(C)** $\frac{\cos^2\theta}{2} + C$ $\qquad = \frac{\sin^2\theta}{2} + C$

(D) $\frac{1}{2}\sin 2\theta + C$ **(E)** $\cos 2\theta + C$

24. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx =$

(A) $-2\cos^{1/2} x + C$ **(B)** $-\cos\sqrt{x} + C$ **(C)** $-2\cos\sqrt{x} + C$

(D) $\frac{3}{2}\sin^{3/2} x + C$ **(E)** $\frac{1}{2}\cos\sqrt{x} + C$

25. $\int t\cos(2t)^2\, dt =$

(A) $\frac{1}{8}\sin(4t^2) + C$ **(B)** $\frac{1}{2}\cos^2(2t) + C$ **(C)** $-\frac{1}{8}\sin(4t^2) + C$

(D) $\frac{1}{4}\sin(2t)^2 + C$ **(E)** none of these

26. $\int \cos^2 2x\, dx =$

(A) $\frac{x}{2} + \frac{\sin 4x}{8} + C$ **(B)** $\frac{x}{2} - \frac{\sin 4x}{8} + C$ **(C)** $\frac{x}{4} + \frac{\sin 4x}{4} + C$

(D) $\frac{x}{4} + \frac{\sin 4x}{16} + C$ **(E)** $\frac{1}{4}(x + \sin 4x) + C$

27. $\frac{1}{2}\int \sin 2\theta\, 2\, d\theta =$ $\quad u = 2\theta$, $du = 2\,d\theta$ $\qquad \frac{1}{2}\int \sin u\, du = \frac{1}{2}(-\cos u) + C$

(A) $\frac{1}{2}\cos 2\theta + C$ **(B)** $-2\cos 2\theta + C$ **(C)** $-\sin^2\theta + C$ $\qquad = -\frac{1}{2}\cos 2\theta + C$

(D) $\cos^2\theta + C$ **(E)** $-\frac{1}{2}\cos 2\theta + C$

BC ONLY

28. $\int x\cos x\, dx =$

(A) $x\sin x + C$ **(B)** $x\sin x + \cos x + C$ **(C)** $x\sin x - \cos x + C$

(D) $\cos x - x\sin x + C$ **(E)** $\frac{x^2}{2}\sin x + C$

29. $\int \frac{du}{\cos^2 3u} =$

(A) $-\frac{\sec 3u}{3} + C$ **(B)** $\tan 3u + C$ **(C)** $u + \frac{\sec 3u}{3} + C$

(D) $\frac{1}{3}\tan 3u + C$ **(E)** $\frac{1}{3\cos 3u} + C$

$u = 1 + \sin x$

$du = \cos x\, dx$

$\int \frac{1}{u^{1/2}}\, du$

$\int u^{-1/2}\, du$

$\frac{2u^{1/2}}{1}$

30. $\int \frac{\cos x\, dx}{\sqrt{1+\sin x}} =$ 36.

(A) $-\frac{1}{2}(1+\sin x)^{1/2}+C$

(B) $\ln\sqrt{1+\sin x}+C$

(C) $2\sqrt{1+\sin x}+C$ 37.

(D) $\ln|1+\sin x|+C$

(E) $\frac{2}{3(1+\sin x)^{3/2}}+C$

31. $\int \frac{\cos(\theta-1)\, d\theta}{\sin^2(\theta-1)} =$ 38.

(A) $2\ln \sin|\theta-1|+C$ **(B)** $-\csc(\theta-1)+C$ **(C)** $-\frac{1}{3}\sin^{-3}(\theta-1)+C$

(D) $-\cot(\theta-1)+C$ **(E)** $\csc(\theta-1)+C$

32. $\int \sec\frac{t}{2}\, dt =$ 39.

(A) $\ln\left|\sec\frac{t}{2}+\tan\frac{t}{2}\right|+C$ **(B)** $2\tan^2\frac{t}{2}+C$ **(C)** $2\ln\cos\frac{t}{2}+C$

(D) $\ln|\sec t+\tan t|+C$ **(E)** $2\ln\left|\sec\frac{t}{2}+\tan\frac{t}{2}\right|+C$

40.

33. $\int \frac{\sin 2x\, dx}{\sqrt{1+\cos^2 x}} =$

(A) $-2\sqrt{1+\cos^2 x}+C$ **(B)** $\frac{1}{2}\ln(1+\cos^2 x)+C$

(C) $\sqrt{1+\cos^2 x}+C$ **(D)** $-\ln\sqrt{1+\cos^2 x}+C$ 41.

(E) $2\ln|\sin x|+C$

34. $\int \sec^{3/2} x \tan x\, dx =$

(A) $\frac{2}{5}\sec^{5/2} x+C$ **(B)** $-\frac{2}{3}\cos^{-3/2} x+C$ **(C)** $\sec^{3/2} x+C$ 42.

(D) $\frac{2}{3}\sec^{3/2} x+C$ **(E)** none of these

35. $\int \tan\theta\, d\theta =$

(A) $-\ln|\sec\theta|+C$ **(B)** $\sec^2\theta+C$ **(C)** $\ln|\sin\theta|+C$

(D) $\sec\theta+C$ **(E)** $-\ln|\cos\theta|+C$

$u = 2x$
$du = 2\,dx$

$\frac{1}{2}\int \csc^2 u\, du$

$\frac{1}{2}\cot u + C$

$-\frac{1}{2}\cot 2x + C$

36. $\frac{1}{2}\int \frac{2\,dx}{\sin^2 2x} =$

(A) $\frac{1}{2}\csc 2x \cot 2x + C$ **(B)** $-\frac{2}{\sin 2x} + C$ **(C)** $-\frac{1}{2}\cot 2x + C$
(D) $-\cot x + C$ **(E)** $-\csc 2x + C$

37. $\int \frac{\tan^{-1} y}{1 + y^2}\, dy =$

(A) $\sec^{-1} y + C$ **(B)** $(\tan^{-1} y)^2 + C$ **(C)** $\ln(1 + y^2) + C$
(D) $\ln(\tan^{-1} y) + C$ **(E)** none of these

38. $\int \sin 2\theta \cos \theta\, d\theta =$

(A) $-\frac{2}{3}\cos^3 \theta + C$ **(B)** $\frac{2}{3}\cos^3 \theta + C$ **(C)** $\sin^2 \theta \cos \theta + C$

(D) $\cos^3 \theta + C$ **(E)** none of these

1) + C

$u = 1 - \cos 2t$
$du = \sin 2t \cdot 2$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|1 - \cos 2t| + C$

39. $\frac{1}{2}\int \frac{2\sin 2t}{1 - \cos 2t}\, dt =$

+ C

(A) $\frac{2}{(1 - \cos 2t)^2} + C$ **(B)** $-\ln|1 - \cos 2t| + C$ **(C)** $\ln\sqrt{|1 - \cos 2t|} + C$

(D) $\sqrt{1 - \cos 2t} + C$ **(E)** $2\ln|1 - \cos 2t| + C$

40. $\int \cot 2u\, du =$

(A) $\ln|\sin u| + C$ **(B)** $\frac{1}{2}\ln|\sin 2u| + C$ **(C)** $-\frac{1}{2}\csc^2 2u + C$

(D) $-\sec 2u + C$ **(E)** $2\ln|\sin 2u| + C$

41. $\int \frac{e^x}{e^x - 1}\, dx =$

(A) $x + \ln|e^x - 1| + C$ **(B)** $x - e^x + C$ **(C)** $x - \frac{1}{(e^x - 1)^2} + C$

(D) $1 + \frac{1}{e^x - 1} + C$ **(E)** $\ln|e^x - 1| + C$

BC ONLY

42. $\int \frac{x - 1}{x(x - 2)}\, dx =$

(A) $\frac{1}{2}\ln|x| + \ln|x - 2| + C$ **(B)** $\frac{1}{2}\ln\left|\frac{x - 2}{x}\right| + C$

(C) $\ln|x - 2| + \ln|x| + C$ **(D)** $\frac{1}{2}\ln|x(x - 2)| + C$

(E) none of these

$u = x^2$
$du = 2x\,dx$

#43
$\frac{1}{2}\int e^u\,du$
$\frac{1}{2}e^{x^2} + C$

43. $\frac{1}{2}\int xe^{x^2}\,dx =$

(A) $\frac{1}{2}e^{x^2} + C$ (B) $e^{x^2}(2x^2+1) + C$ (C) $2e^{x^2} + C$

(D) $e^{x^2} + C$ (E) $\frac{1}{2}e^{x^2+1} + C$

44. $\int \cos\theta\, e^{\sin\theta}\,d\theta =$

(A) $e^{\sin\theta+1} + C$ (B) $e^{\sin\theta} + C$ (C) $-e^{\sin\theta} + C$

(D) $e^{\cos\theta} + C$ (E) $e^{\sin\theta}(\cos\theta - \sin\theta) + C$

#45 $\frac{1}{2}\int \sin u\,du$
$-\frac{1}{2}\cos u + C$

45. $\frac{1}{2}\int e^{2\theta}\sin e^{2\theta}\,d\theta =$

$u = e^{2\theta}$
$du = e^{2\theta}\cdot 2\,d\theta$

(A) $\cos e^{2\theta} + C$ (B) $2e^{4\theta}(\cos e^{2\theta} + \sin e^{2\theta}) + C$ (C) $-\frac{1}{2}\cos e^{2\theta} + C$

(D) $-2\cos e^{2\theta} + C$ (E) none of these

46. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$

(A) $2\sqrt{x}(e^{\sqrt{x}} - 1) + C$ (B) $2e^{\sqrt{x}} + C$ (C) $\frac{e^{\sqrt{x}}}{2}\left(\frac{1}{x} + \frac{1}{x\sqrt{x}}\right) + C$

(D) $\frac{1}{2}e^{\sqrt{x}} + C$ (E) none of these

BC ONLY

47. $\int xe^{-x}\,dx =$

(A) $e^{-x}(1-x) + C$ (B) $\frac{e^{1-x}}{1-x} + C$ (C) $-e^{-x}(x+1) + C$

(D) $-\frac{x^2}{2}e^{-x} + C$ (E) $e^{-x}(x+1) + C$

48. $\int x^2e^x\,dx =$

(A) $e^x(x^2+2x) + C$ (B) $e^x(x^2-2x-2) + C$ (C) $e^x(x^2-2x+2) + C$

(D) $e^x(x-1)^2 + C$ (E) $e^x(x+1)^2 + C$

49. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx =$

$u = e^x - e^{-x}$
$du = e^x - e^{-x}(-1) = (e^x + e^{-x})\,dx$

$\ln|e^x - e^{-x}| + C$

(A) $x - \ln|e^x - e^{-x}| + C$ (B) $x + 2\ln|e^x - e^{-x}| + C$

(C) $-\frac{1}{2}(e^x - e^{-x})^{-2} + C$ (D) $\ln|e^x - e^{-x}| + C$

(E) $\ln(e^x + e^{-x}) + C$

50. $\int \frac{e^x}{1+e^{2x}}\,dx =$

(A) $\tan^{-1} e^x + C$ **(B)** $\frac{1}{2}\ln(1+e^{2x}) + C$ **(C)** $\ln(1+e^{2x}) + C$

(D) $\frac{1}{2}\tan^{-1} e^x + C$ **(E)** $2\tan^{-1} e^x + C$

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51. $\int \frac{\ln v\,dv}{v} =$

$u = \ln v$

$du = \frac{1}{v}\,dv$

$\int u\,du = \frac{u^2}{2} + c = \frac{1}{2}(\ln v)^2 + c$

(A) $\ln|\ln v| + C$ **(B)** $\ln\frac{v^2}{2} + C$ **(C)** $\frac{1}{2}(\ln v)^2 + C$

(D) $2\ln v + C$ **(E)** $\frac{1}{2}\ln v^2 + C$

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52. $\int \frac{\ln\sqrt{x}}{x}\,dx =$

$u = \ln(x^{1/2}) = \frac{1}{2}\ln x$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\frac{1}{2}\int \frac{\ln x}{x}$

$\frac{1}{2}\int u\,du$

$\frac{1}{2}\frac{u^2}{2} + c$

(A) $\frac{\ln^2\sqrt{x}}{\sqrt{x}} + C$ **(B)** $\ln^2 x + C$ **(C)** $\frac{1}{2}\ln|\ln x| + C$

(D) $\frac{(\ln\sqrt{x})^2}{2} + C$ **(E)** $\frac{1}{4}\ln^2 x + C$

BC ONLY

53. $\int x^3 \ln x\,dx =$

(A) $x^2(3\ln x + 1) + C$ **(B)** $\frac{x^4}{16}(4\ln x - 1) + C$ **(C)** $\frac{x^4}{4}(\ln x - 1) + C$

(D) $3x^2\left(\ln x - \frac{1}{2}\right) + C$ **(E)** none of these

54. $\int \ln\eta\,d\eta =$

(A) $\frac{1}{2}\ln^2\eta + C$ **(B)** $\eta(\ln\eta - 1) + C$ **(C)** $\frac{1}{2}\ln\eta^2 + C$

(D) $\ln\eta(\eta - 1) + C$ **(E)** $\eta\ln\eta + \eta + C$

55. $\int \ln x^3\,dx =$

(A) $\frac{3}{2}\ln^2 x + C$ **(B)** $3x(\ln x - 1) + C$ **(C)** $3\ln x(x-1) + C$

(D) $\frac{3x\ln^2 x}{2} + C$ **(E)** none of these

BC ONLY

56. $\int \frac{\ln y}{y^2}\, dy =$

(A) $\frac{1}{y}(1-\ln y)+C$ (B) $\frac{1}{2y}\ln^2 y+C$ (C) $-\frac{1}{3y^3}(4\ln y+1)+C$

(D) $-\frac{1}{y}(\ln y+1)+C$ (E) $\frac{\ln y}{y}-\frac{1}{y}+C$

57. $\int \frac{dv}{v\ln v} =$

(A) $\frac{1}{\ln v^2}+C$ (B) $-\frac{1}{\ln^2 v}+C$ (C) $-\ln|\ln v|+C$

(D) $\ln\frac{1}{v}+C$ (E) $\ln|\ln v|+C$

58. $\int \frac{y-1}{y+1}\, dy =$

(A) $y-2\ln|y+1|+C$ (B) $1-\frac{2}{y+1}+C$ (C) $\ln\frac{|y|}{(y+1)^2}+C$

(D) $1-2\ln|y+1|+C$ (E) $\ln\left|\frac{e^y}{y+1}\right|+C$

59. $\int \frac{dx}{x^2+2x+2} =$

(A) $\ln(x^2+2x+2)+C$ (B) $\ln|x+1|+C$ (C) $\arctan(x+1)+C$

(D) $\frac{1}{\frac{1}{3}x^3+x^2+2x}+C$ (E) $-\frac{1}{x}+\frac{1}{2}\ln|x|+\frac{x}{2}+C$

#60

$\int(x^{1}-x^{1/2})\,dx$

$\frac{x^2}{2}-\frac{2x^{3/2}}{3}+C$

60. $\int \sqrt{x}(\sqrt{x}-1)\,dx =$ $\int x^{1/2}(x^{1/2}-1)$

(A) $2(x^{3/2}-x)+C$ (B) $\frac{x^2}{2}-x+C$ (C) $\frac{1}{2}(\sqrt{x}-1)^2+C$

(D) $\frac{1}{2}x^2-\frac{2}{3}x^{3/2}+C$ (E) $x-2\sqrt{x}+C$

BC ONLY

61. $\int e^{\theta}\cos\theta\, d\theta =$

(A) $e^{\theta}(\cos\theta-\sin\theta)+C$
(B) $e^{\theta}\sin\theta+C$
(C) $\frac{1}{2}e^{\theta}(\sin\theta+\cos\theta)+C$
(D) $2e^{\theta}(\sin\theta+\cos\theta)+C$
(E) $\frac{1}{2}e^{\theta}(\sin\theta-\cos\theta)+C$

C

62. $\int \frac{(1-\ln t)^2}{t}\, dt =$

(A) $\frac{1}{3}(1-\ln t)^3 + C$ (B) $\ln t - 2\ln^2 t + \ln^3 t + C$ (C) $-2(1-\ln t) + C$

(D) $\ln t - \ln^2 t + \frac{\ln t^3}{3} + C$ (E) $-\frac{(1-\ln t)^3}{3} + C$

63. $\int u \sec^2 u\, du =$

BC ONLY

(A) $u \tan u + \ln|\cos u| + C$ (B) $\frac{u^2}{2}\tan u + C$ (C) $\frac{1}{2}\sec u \tan u + C$

(D) $u \tan u - \ln|\sin u| + C$ (E) $u \sec u - \ln|\sec u + \tan u| + C$

64. $\int \frac{2x+1}{4+x^2}\, dx =$

CHALLENGE

(A) $\ln(x^2+4) + C$ (B) $\ln(x^2+4) + \tan^{-1}\frac{x}{2} + C$ (C) $\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

(D) $\ln(x^2+4) + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$ (E) none of these

65. $\int \frac{1-x}{\sqrt{1-x^2}}\, dx =$

CHALLENGE

(A) $\sqrt{1-x^2} + C$ (B) $\sin^{-1} x + C$

(C) $\frac{1}{2}\ln\sqrt{1-x^2} + C$ (D) $\sin^{-1} x + \sqrt{1-x^2} + C$

(E) $\sin^{-1} x + \frac{1}{2}\ln\sqrt{1-x^2} + C$

66. $\int \frac{2x-1}{\sqrt{4x-4x^2}}\, dx =$

CHALLENGE

(A) $4\ln\sqrt{4x-4x^2} + C$ (B) $\sin^{-1}(1-2x) + C$

(C) $\frac{1}{2}\sqrt{4x-4x^2} + C$ (D) $-\frac{1}{4}\ln(4x-4x^2) + C$

(E) $-\frac{1}{2}\sqrt{4x-4x^2} + C$

67. $\int \frac{e^{2x}}{1+e^x}\, dx =$

CHALLENGE

(A) $\tan^{-1} e^x + C$ (B) $e^x - \ln(1+e^x) + C$ (C) $e^x - x + \ln\left|1+e^x\right| + C$

(D) $e^x + \frac{1}{(e^x+1)^2} + C$ (E) none of these

68. $\int \frac{\cos\theta}{1+\sin^2\theta}\, d\theta =$

(A) $\sec\theta\tan\theta + C$ (B) $\sin\theta - \csc\theta + C$ (C) $\ln(1+\sin^2\theta) + C$

(D) $\tan^{-1}(\sin\theta) + C$ (E) $-\frac{1}{(1+\sin^2\theta)^2} + C$

BC ONLY

69. $\int \arctan x\, dx =$

(A) $\arctan x + C$

(B) $x\arctan x - \ln(1+x^2) + C$

(C) $x\arctan x + \ln(1+x^2) + C$

(D) $x\arctan x + \frac{1}{2}\ln(1+x^2) + C$

(E) $x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

CHALLENGE

70. $\int \frac{dx}{1-e^x} =$

(A) $-\ln|1-e^x| + C$ (B) $x - \ln|1-e^x| + C$ (C) $\frac{1}{(1-e^x)^2} + C$

(D) $e^{-x}\ln|1+e^x| + C$ (E) none of these

71. $\int \frac{(2-y)^2}{4\sqrt{y}}\, dy =$

(A) $\frac{1}{6}(2-y)^3\sqrt{y} + C$

(B) $2\sqrt{y} - \frac{2}{3}y^{3/2} + \frac{8}{5}y^{5/2} + C$

(C) $\ln|y| - y + 2y^2 + C$

(D) $2y^{1/2} - \frac{2}{3}y^{3/2} + \frac{1}{10}y^{5/2} + C$

(E) none of these

72. $\int e^{2\ln u}\, du =$

(A) $\frac{1}{3}e^{u^3} + C$ (B) $e^{u^3/3} + C$ (C) $\frac{1}{3}u^3 + C$

(D) $\frac{2}{u}e^{2\ln u} + C$ (E) $e^{1+2\ln u} + C$

73. $\int \frac{dy}{y(1+\ln y^2)} =$

(A) $\frac{1}{2}\ln|1+\ln y^2| + C$ (B) $-\frac{1}{(1+\ln y^2)^2} + C$

(C) $\ln|y| + \frac{1}{2}\ln|\ln y| + C$ (D) $\tan^{-1}(\ln|y|) + C$ (E) none of these

74. $\int (\tan\theta - 1)^2\, d\theta =$

+ C

(A) $\sec\theta + \theta + 2\ln|\cos\theta| + C$ (B) $\tan\theta + 2\ln|\cos\theta| + C$

(C) $\tan\theta - 2\sec^2\theta + C$ (D) $\sec\theta + \theta - \tan^2\theta + C$

(E) $\tan\theta - 2\ln|\cos\theta| + C$

CHALLENGE

75. $\int \frac{d\theta}{1 + \sin\theta} =$

(A) $\sec\theta - \tan\theta + C$ (B) $\ln(1 + \sin\theta) + C$

(C) $\ln|\sec\theta + \tan\theta| + C$ (D) $\theta + \ln|\csc\theta - \cot\theta| + C$

(E) none of these

CHALLENGE

$V(0) = 0$

76. A particle starting at rest at $t = 0$ moves along a line so that its acceleration at time t is $12t$ ft/sec². How much distance does the particle cover during the first 3 sec?

(A) 16 ft (B) 32 ft (C) 48 ft (D) 54 ft (E) 108 ft

$a(t) = 12t$

$V(t) = \int 12t$

$V(t) = 12\frac{t^2}{2} + C$

$0 = C$

$\int v(t) = \int_0^3 6t^2$

$x(t) = 6\left[\frac{t^3}{3}\right]\Big|_0^3$

$= 6[9 - 0]$

$= 54$

77. The equation of the curve whose slope at point (x, y) is $x^2 - 2$ and which contains the point $(1, -3)$ is

(A) $y = \frac{1}{3}x^3 - 2x$ (B) $y = 2x - 1$ (C) $y = \frac{1}{3}x^3 - \frac{10}{3}$

(D) $y = \frac{1}{3}x^3 - 2x - \frac{4}{3}$ (E) $3y = x^3 - 10$

$V(0) = 3$

78. A particle moves along a line with acceleration $2 + 6t$ at time t. When $t = 0$, its velocity equals 3 and it is at position $s = 2$. When $t = 1$, it is at position $s =$

$S(0) = 2$

(A) 2 (B) 5 (C) 6 (D) 7 (E) 8

$V(t) = \int(2 + 6t) = 2t + 3t^2 + C$

$3 = 2(0) + 3(0) + C$

$C = 3$

79. Find the acceleration (in ft/sec²) needed to bring a particle moving with a velocity of 75 ft/sec to a stop in 5 sec.

(A) −3 (B) −6 (C) −15 (D) −25 (E) −30

$S(t) = \int 2t + 3t^2 + 3$

$= t^2 + t^3 + 3t + C$

$S(0) = 2$

$2 = C$

80. $\int \frac{x^2}{x^2 - 1}\, dx =$

BC ONLY

(A) $x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$ (B) $\ln|x^2 - 1| + C$ (C) $x + \tan^{-1}x + C$

(D) $x + \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C$ (E) $1 + \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C$

CHALLENGE

$S(1) = 1 + 1 + 3 + 2$

$= 7$

Answer Key

1. C	**17.** E	**33.** A	**49.** D	**65.** D
2. E	**18.** D	**34.** D	**50.** A	**66.** E
3. A	**19.** A	**35.** E	**51.** C	**67.** B
4. D	**20.** D	**36.** C	**52.** E	**68.** D
5. E	**21.** E	**37.** E	**53.** B	**69.** E
6. B	**22.** E	**38.** A	**54.** B	**70.** B
7. A	**23.** B	**39.** C	**55.** B	**71.** D
8. E	**24.** C	**40.** B	**56.** D	**72.** C
9. D	**25.** A	**41.** E	**57.** E	**73.** A
10. A	**26.** A	**42.** D	**58.** A	**74.** B
11. D	**27.** E	**43.** A	**59.** C	**75.** E
12. C	**28.** B	**44.** B	**60.** D	**76.** D
13. B	**29.** D	**45.** C	**61.** C	**77.** D
14. C	**30.** C	**46.** B	**62.** E	**78.** D
15. B	**31.** B	**47.** C	**63.** A	**79.** C
16. A	**32.** E	**48.** C	**64.** D	**80.** A

Answers Explained

All the references in parentheses below are to the basic integration formulas on pages 217 and 218. In general, if u is a function of x, then $du = u'(x)\,dx$.

1. (C) Use, first, formula (2), then (3), replacing u by x.

2. (E) Hint: Expand. $\int \left(x^2 - 1 + \frac{1}{4x^2}\right) dx = \frac{x^3}{3} - x - \frac{1}{4x} + C$.

3. (A) By formula (3), with $u = 4 - 2t$ and $n = \frac{1}{2}$,

$$\int \sqrt{4-2t}\,dx = -\frac{1}{2}\int \sqrt{4-2t}\cdot(-2dt) = -\frac{1}{2}\frac{(4-2t)^{3/2}}{3/2} + C.$$

4. (D) Rewrite: $-\frac{1}{3}\int (2-3x)^5(-3\,dx)$

5. (E) Rewrite:

$$\int (2y-3y^2)^{-1/2}(1-3y)\,dy = \frac{1}{2}\int (2y-3y^2)^{-1/2}(2-6y)\,dy.$$

Use (3).

6. (B) Rewrite:

$$\frac{1}{3}\int (2x-1)^{-2}\,dx = \frac{1}{3}\cdot\frac{1}{2}\int (2x-1)^{-2}\cdot 2\,dx.$$

Using (3) yields $-\frac{1}{6(2x-1)} + C$.

Chapter Summary

In this chapter, we have reviewed definite integrals, starting with the Fundamental Theorem of Calculus. We've looked at techniques for evaluating definite integrals algebraically, numerically, and graphically. We've reviewed Riemann sums, including the left, right, and midpoint approximations as well as the trapezoid rule. We have also looked at the average value of a function.

This chapter also reviewed integrals based on parametrically defined functions, a BC Calculus topic.

Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

1. $\int_{-1}^{1} (x^2 - x - 1)\,dx =$ $\left(\frac{x^3}{3} - \frac{x^2}{2} - x\right)\Big|_{-1}^{1} = \left[\left(\frac{1}{3} - \frac{1}{2} - 1\right) - \left(-\frac{1}{3} - \frac{1}{2} + 1\right)\right]$

(A) $\frac{2}{3}$ (B) 0 (C) $-\frac{4}{3}$ (D) -2 (E) -1

$= \frac{1}{3} - \cancel{\frac{1}{2}} - 1 + \frac{1}{3} + \cancel{\frac{1}{2}} - 1$

$= \frac{2}{3} - 2 = \frac{2}{3} - \frac{6}{3} = -4/3$

2. $\int_{1}^{2} \frac{3x - 1}{3x}\,dx =$ $\int_1^2 \frac{3x}{3x}dx - \int_1^2 \frac{1}{3x}\,dx = \int_1^2 dx - \frac{1}{3}\int \frac{1}{x}\,dx = \left(x - \frac{1}{3}\ln x\right)\Big|_1^2$

(A) $\frac{3}{4}$ (B) $1 - \frac{1}{3}\ln 2$ (C) $1 - \ln 2$ (D) $-\frac{1}{3}\ln 2$ (E) 1

$\left(2 - \frac{1}{3}\ln 2\right) - \left(1 - \frac{1}{3}\ln 1\right)$

$2 - \frac{1}{3}\ln 2 - 1 + 0$

$1 - \frac{1}{3}\ln 2$

3. $\int_{0}^{3} \frac{dt}{\sqrt{4 - t}} =$ $u = 4 - t$, $du = -dt$ $\quad -\int \frac{du}{u^{1/2}} = -\int u^{-1/2}\,du$

(A) 1 (B) -2 (C) 4 (D) -1 (E) 2

$= -\frac{2u^{1/2}}{1}\Big|_0^3 = -\frac{2\sqrt{4-t}}{1}\Big|_0^3$

$= -2\left(\sqrt{4-3} - \sqrt{4-0}\right)$

$= -2(1 - 2)$

$= -2(-1) = \boxed{2}$

4. $\int_{-1}^{0} \sqrt{3u + 4}\,du =$

(A) 2 (B) $\frac{14}{9}$ (C) $\frac{14}{3}$ (D) 6 (E) $\frac{7}{2}$

5. $\frac{1}{2}\int_{2}^{3} \frac{2\,dy}{2y - 3} =$ $u = 2y - 3$, $du = 2dy$ $\quad \frac{1}{2}\int_2^3 \frac{1}{u}\,du$

(A) $\ln 3$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\frac{16}{9}$ (D) $\ln\sqrt{3}$ (E) $\sqrt{3} - 1$

$\to \frac{1}{2}\ln u\Big|_2^3 = \frac{1}{2}\left[\ln(2y-3)\right]\Big|_2^3$

$= \frac{1}{2}\left[\ln 3 - \ln 1\right]$

$= \frac{1}{2}\ln 3$

$= \ln\sqrt{3}$

6. $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4 - x^2}}\,dx =$

(A) 1 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) -1 (E) 2

#7 $u = 2t - 1$
$du = 2\,dt$
$\frac{1}{2}\int u^3\,du$
$\frac{1}{2}\frac{u^4}{4}\Big|_0^1$
$\frac{1}{8}(2t-1)^4\Big|_0^1$
$\frac{1}{8}\left[(2-1)^4 - (0-1)^4\right] = \frac{1}{8}[1-1]$

7. $\frac{1}{2}\int_0^1 (2t-1)^3 (2)\,dt =$ 15.

(A) $\frac{1}{4}$ (B) 6 (C) $\frac{1}{2}$ (D) 0 (E) 4

8. $\int_4^9 \frac{2+x}{2\sqrt{x}}\,dx =$ 16.

(A) $\frac{25}{3}$ (B) $\frac{41}{3}$ (C) $\frac{100}{3}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$

9. $\int_{-3}^{3} \frac{dx}{9+x^2} = \int \frac{\frac{1}{9}dx}{9(1+(\frac{x}{3})^2)} = \frac{3}{9}\int \frac{du}{1+u^2}$ 17.

$u = x/3$ $du = \frac{1}{3}dx$

(A) $\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{6}$ (D) $-\frac{\pi}{2}$ (E) $\frac{\pi}{3}$

#9 $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\Big|_{-3}^{3}$
$\frac{1}{3}\left[\tan^{-1}(1) - \tan^{-1}(-1)\right]$
$\frac{1}{3}\left[\pi/4 - (-\pi/4)\right]$
$\frac{1}{3}\cdot\frac{\pi}{2} = \pi/6$

10. $-\int_0^1 e^{-x}\,dx =$ $u = -x$ $du = -dx$ 18.

(A) $\frac{1}{e} - 1$ (B) $1 - e$ (C) $-\frac{1}{e}$ (D) $1 - \frac{1}{e}$ (E) $\frac{1}{e}$

#10 $-\int_0^1 e^u\,du = -e^{-x}\Big|_0^1$
$= [-e^{-1} + e^0]$
$1 - \frac{1}{e}$

11. $\frac{1}{2}\int_0^1 x e^{x^2}\,dx =$ $u = x^2$ $du = 2x\,dx$

(A) $e - 1$ (B) $\frac{1}{2}(e-1)$ (C) $2(e-1)$ (D) $\frac{e}{2}$ (E) $\frac{e}{2} - 1$ 19.

#11 $\frac{1}{2}\int_0^1 e^u\,du$
$\frac{1}{2}\left(e^{x^2}\right)\Big|_0^1 = \frac{1}{2}(e-1)$
$\frac{1}{2}e^1 - \frac{1}{2}e^0$

12. $\frac{1}{2}\int_0^{\pi/4} \sin 2\theta\,(2)\,d\theta =$ $u = 2\theta$ $du = 2\,d\theta$

(A) 2 (B) $\frac{1}{2}$ (C) -1 (D) $-\frac{1}{2}$ (E) -2 20.

#12 $\frac{1}{2}\int_0^{\pi/4} \sin u\,du$
$= \frac{1}{2}[\sin 2\theta]_0^{\pi/4}$
$= \frac{1}{2}[\sin \frac{2\pi}{4} - \sin 0] = \frac{1}{2}(1) = \frac{1}{2}$

13. $-\int_1^2 \frac{-dz}{3-z} =$ $u = 3 - z$ $du = -1\,dz$

#13 $-\int_1^2 \frac{du}{u} = -\ln|3-z|\Big|_1^2$
$-\ln|3-2| + \ln|3-1|$
$-\ln 1 + \ln 2$

(A) $-\ln 2$ (B) $\frac{3}{4}$ (C) $2(\sqrt{2} - 1)$ (D) $\frac{1}{2}\ln 2$ (E) $\ln 2$ 21.

14. If we let $x = 2\sin\theta$, then $\int_1^2 \frac{\sqrt{4-x^2}}{x}\,dx$ is equivalent to

(A) $2\int_0^2 \frac{\cos^2\theta}{\sin\theta}\,d\theta$ (B) $\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin\theta}\,d\theta$ (C) $2\int_{\pi/6}^{\pi/2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$ 22.

(D) $\int_1^2 \frac{\cos\theta}{\sin\theta}\,d\theta$ (E) none of these

$\int \frac{\sqrt{4 - 4\sin^2\theta}}{2\sin\theta}$

$\int_{\pi/6}^{\pi/2} \frac{\cancel{2}\sqrt{1-\sin^2\theta}}{\cancel{2}\sin\theta}$

$\int_{\pi/6}^{\pi/2} \frac{\cos^2\theta}{\sin\theta}$

@ $x = 2$ $2 = 2\sin\theta$
$\sin\theta = 1$
$\theta = \pi/2$

@ $x = 1$ $1 = 2\sin\theta$
$\frac{1}{2} = \sin\theta$
$\theta = \pi/6$

$u = \cos\theta$

$du = -\sin\theta$

15. $-\int_0^{\pi} \cos^2\theta \sin\theta \, d\theta =$ $\quad -\int u^2 du = -\frac{1}{3}[\cos^3\theta]\Big|_0^{\pi} = -\frac{1}{3}[\cos\pi - \cos 0]$

$= -\frac{1}{3}[-1-1]$

$= -\frac{1}{3}[-2]$

$= 2/3$

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{2}{3}$ (E) 0

16. $\int_1^e \frac{\ln x}{x} dx =$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int_1^e u\,du = \frac{u^2}{2} = \frac{[\ln x]^2}{2}\Big|_1^e$

$\frac{[\ln e]^2}{2} - \frac{[\ln 1]^2}{2}$

$\frac{1^2}{2} = \frac{1}{2}$

(A) $\frac{1}{2}$ (B) $\frac{1}{2}(e^2-1)$ (C) 0 (D) 1 (E) $e-1$

BC ONLY

17. $\int_0^1 xe^x \, dx =$

(A) -1 (B) $e+1$ (C) 1 (D) $e-1$ (E) $\frac{1}{2}(e-1)$

$u = 1 + 2\sin\theta$

$du = 2\cos\theta$

18. $\frac{1}{2}\int_0^{\pi/6} \frac{2\cos\theta}{1+2\sin\theta} d\theta =$ $\quad \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u = \frac{1}{2}\ln(1+2\sin\theta)\Big|_0^{\pi/6}$

$= \frac{1}{2}\ln(1+2\sin\frac{\pi}{6}) - \ln(1+2\sin 0)$

$= \frac{1}{2}\ln(1+1) - \frac{1}{2}\ln(1+0)$

$= \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1$

$= \ln 2^{1/2}$

(A) $\ln 2$ (B) $\frac{3}{8}$ (C) $-\frac{1}{2}\ln 2$ (D) $\frac{3}{2}$ (E) $\ln\sqrt{2}$

-1 19. $\int_{\sqrt{2}}^{2} \frac{u}{u^2-1} du =$

(A) $\ln\sqrt{3}$ (B) $\frac{8}{9}$ (C) $\ln\frac{3}{2}$ (D) $\ln 3$ (E) $1-\sqrt{3}$

$u = x^2 - 1$

$du = 2x\,dx$

20. $\frac{1}{2}\int_{\sqrt{2}}^{2} \frac{2\cancel{x}\,du}{(\cancel{x}^2-1)^2} =$ $\quad \frac{1}{2}\int \frac{du}{u^2} = \frac{1}{2}\int u^{-2}du = \frac{1}{2}\left(-\frac{u^{-1}}{1}\right) = -\frac{1}{2u}$

$-\frac{1}{2(x^2-1)}\Big|_{\sqrt{2}}^{2}$

$= -\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$

(A) $-\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) -1 (E) $\frac{1}{3}$

$u = \sin 2x$

$du = \cos 2x \cdot 2\,dx$

n 2 21. $\frac{1}{2}\int_{\pi/12}^{\pi/4} \frac{\cos 2x \, 2dx}{\sin^2 2x} =$ $\quad \frac{1}{2}\int \frac{du}{u^2} = -\frac{1}{2u} = -\frac{1}{2\sin 2x}\Big|_{\pi/12}^{\pi/4}$

$= \left[-\frac{1}{2\sin(\pi/2)} + \frac{1}{2\sin(\pi/6)}\right]$

$= -\frac{1}{2(1)} + \frac{1}{2(\frac{1}{2})} = \frac{1}{2}$

(A) $-\frac{1}{4}$ (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) -1

22. $\int_0^1 \frac{e^{-x}+1}{e^{-x}} dx =$ $\quad \int_0^1\left(\frac{e^{-x}}{e^{-x}} + \frac{1}{e^{-x}}\right)dx = \int_0^1 1\,dx + \int_0^1 e^x dx$

(A) e (B) $2+e$ (C) $\frac{1}{e}$ (D) $1+e$ (E) $e-1$

$\to x\Big|_0^1 + e^x\Big|_0^1 = (1-0) + (e^1 - e^0)$

$= 1 + e^1 - 1$

$= e$

23. $\int_0^1 \frac{e^x}{e^x+1}\,dx =$

(A) $\ln 2$ (B) e (C) $1+e$ (D) $-\ln 2$ (E) $\ln\frac{e+1}{2}$

24. If we let $x = \tan\theta$, then $\int_1^{\sqrt{3}} \sqrt{1+x^2}\,dx$ is equivalent to

(A) $\int_{\pi/4}^{\pi/3} \sec\theta\,d\theta$ (B) $\int_1^{\sqrt{3}} \sec^3\theta\,d\theta$ (C) $\int_{\pi/4}^{\pi/3} \sec^3\theta\,d\theta$

(D) $\int_{\pi/4}^{\pi/3} \sec^2\theta\tan\theta\,d\theta$ (E) $\int_1^{\sqrt{3}} \sec\theta\,d\theta$

25. If the substitution $u = \sqrt{x+1}$ is used, then $\int_0^3 \frac{dx}{x\sqrt{x+1}}$ is equivalent to

(A) $\int_1^2 \frac{du}{u^2-1}$ (B) $\int_1^2 \frac{2\,du}{u^2-1}$ (C) $2\int_0^3 \frac{du}{(u-1)(u+1)}$

(D) $2\int_1^2 \frac{du}{u(u^2-1)}$ (E) $2\int_0^3 \frac{du}{u(u-1)}$

#26 FTC

$\int_8^0 f'(x)dx = f(0)-f(8)$

$= 11-7$

$= 4$

26. The table shows some values of continuous function f and its first derivative. Evaluate $\int_8^0 f'(x)\,dx.$ $= f(0)-f(8)$

(A) $-1/2$ (B) $-3/8$ (C) 3
(D) 4 (E) none of these

x	$f(x)$	$f'(x)$
0	11	3
2	15	2
4	16	–1
6	12	–3
8	7	0

#27 M(3) Midpt 3 rectangles

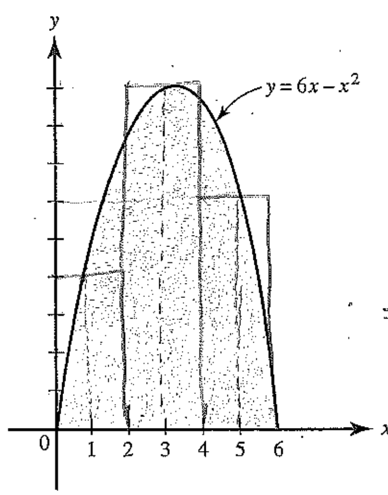
27. Using $M(3)$, we find that the approximate area of the shaded region below is

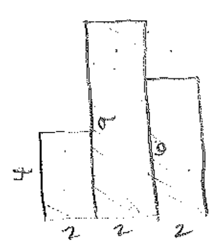
(A) 9 (B) 19 (C) 36 (D) 38 (E) 54

$4(2)+9(2)+6(2)$

$8+18+12$

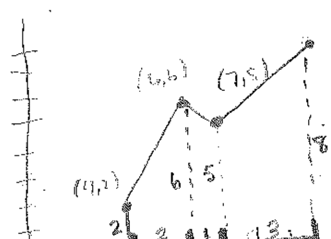
38





#28  $A = \frac{1}{2}b(h_1+h_2)$

Area of Trapezoid



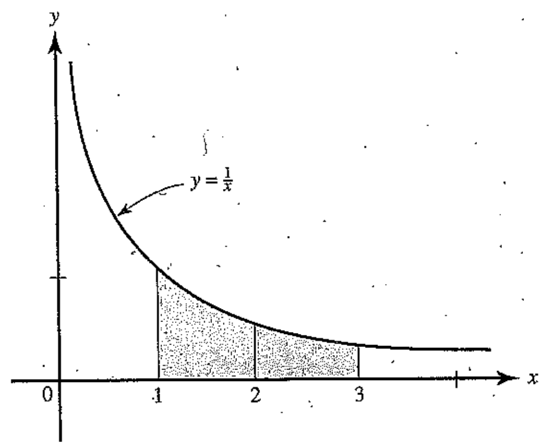
$T(3) = \frac{1}{2}[2(2+6)+1(6+5)+3(5+8)]$

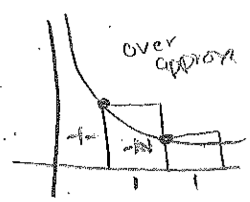
$= \frac{1}{2}[16+11+39] = 33$

28. The graph of a continuous function f passes through the points (4,2), (6,6), (7,5), and (10,8). Using trapezoids, we estimate that $\int_4^{10} f(x)dx \approx$

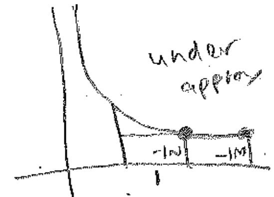
(A) 25 (B) 30 (C) 32 (D) 33 (E) 41

29. The area of the shaded region in the figure is equal exactly to ln 3. If we approximate ln 3 using $L(2)$ and $R(2)$, which inequality follows?





L (2)
$A = 1 + \frac{1}{2}$
$= \frac{3}{2}$ over



R(2)
$A = \frac{1}{2} + \frac{1}{3}$
$= \frac{3}{6} + \frac{2}{6}$
$= \frac{5}{6}$ under

$\int_1^3 \frac{1}{x}\,dx$ exact

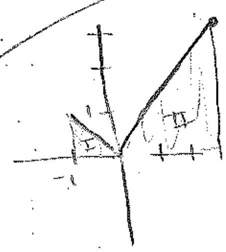
(A) $\frac{1}{2} < \int_1^2 \frac{1}{x}\,dx < 1$ (B) $\frac{1}{3} < \int_1^3 \frac{1}{x}\,dx < 2$ (C) $\frac{1}{2} < \int_0^2 \frac{1}{x}\,dx < 2$

(D) $\frac{1}{3} < \int_2^3 \frac{1}{x}\,dx < \frac{1}{2}$ (E) $\frac{5}{6} < \int_1^3 \frac{1}{x}\,dx < \frac{3}{2}$

30. Let $A = \int_0^1 \cos x\,dx$. We estimate A using the L, R, and T approximations with $n = 100$ subintervals. Which is true?

(A) $L < A < T < R$
(B) $L < T < A < R$
(C) $R < A < T < L$
(D) $R < T < A < L$
(E) The order cannot be determined.

$A = \mathrm{I} + \mathrm{II}$
$= \frac{1}{2}(1)(1) + \frac{1}{2}(3)(3)$
$= \frac{1}{2} + \frac{9}{2}$
$= 5$

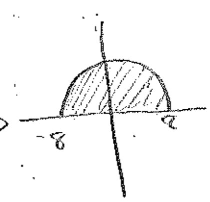


31. $\int_{-1}^{3} |x|\,dx =$

(A) $\frac{7}{2}$ (B) 4 (C) $\frac{9}{2}$ (D) 5 (E) $\frac{11}{2}$

32. $\int_{-3}^{2} |x+1|\,dx =$

(A) $\frac{5}{2}$ (B) $\frac{7}{2}$ (C) 5 (D) $\frac{11}{2}$ (E) $\frac{13}{2}$



circle radius 8

33. The average value of $y = \sqrt{64 - x^2}$ on its domain is

(A) 2 (B) 4 (C) 2π (D) 4π (E) none of these

Area of ½ circle w/ r=8

$A = \frac{1}{2}(\pi r^2)$
$= \frac{1}{2}(\pi)(64)$
$= 32\pi$

Avg Value $= \frac{\int y\,dy}{8-(-8)} = \frac{\text{Area}}{16} = \frac{32\pi}{16} = 2\pi$

#34 $\dfrac{\int_{\pi/3}^{\pi/2} \cos x \, dx}{\pi/2 - \pi/3}$

$[\sin \pi/2 - \sin \pi/3]$ / $\frac{3\pi}{6} - \frac{2\pi}{6}$

$\dfrac{1-\sqrt{3}/2}{\frac{\pi}{6}}$ keep mult flip

$\dfrac{(2-\sqrt{3})}{2} \cdot \dfrac{6}{\pi} = (2-\sqrt{3})\dfrac{3}{\pi}$

34. The average value of $\cos x$ over the interval $\frac{\pi}{3} \leqq x \leqq \frac{\pi}{2}$ is

(A) $\frac{3}{\pi}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3(2-\sqrt{3})}{\pi}$ **(D)** $\frac{3}{2\pi}$ **(E)** $\frac{2}{3\pi}$

35. The average value of $\csc^2 x$ over the interval from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{4}$ is

(A) $\frac{3\sqrt{3}}{\pi}$ **(B)** $\frac{\sqrt{3}}{\pi}$ **(C)** $\frac{12}{\pi}(\sqrt{3}-1)$

(D) $3\sqrt{3}$ **(E)** $3(\sqrt{3}-1)$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

#36 $\dfrac{\int_0^5 f \, dx}{5-0}$

$= \dfrac{\text{Area}}{5}$

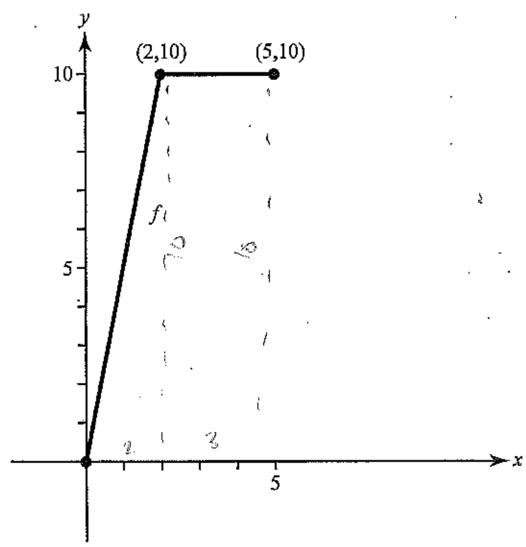
$= \dfrac{\frac{1}{2}(2)(10) + 3(10)}{5}$

$= \dfrac{10+30}{5}$

$= \dfrac{40}{5}$

$= 8$

36. Find the average value of function f, as shown in the graph below, on the interval [0,5].



(A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 8

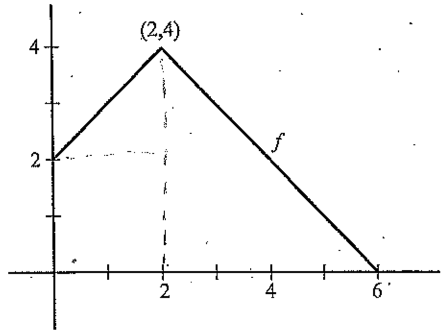
37. The integral $\int_{-4}^{4} \sqrt{16-x^2}\, dx$ gives the area of

(A) a circle of radius 4
(B) a semicircle of radius 4
(C) a quadrant of a circle of radius 4
(D) an ellipse whose semimajor axis is 4
(E) none of these

38. $\int_0^{\pi/4} \sqrt{1-\cos 2\alpha}\, d\alpha =$

(A) 0.25 **(B)** 0.414 **(C)** 1.000 **(D)** 1.414 **(E)** 2.000

Use the graph of function *f*, shown below, for questions 39–42.



39. In which of these intervals is there a value *c* for which $f(c)$ is the average value of *f* over the interval $[0,6]$?

I. [0,2]
II. [2,4]
III. [4,6]

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) none of these, because *f* is not differentiable on [0,6]

40. $\frac{1}{3}\int_0^2 f'(3x)\,3dx =$ $\quad u = f(3x)$, $du = f'(3x)\,3\,dx$ $\quad \frac{1}{3}\int du = \frac{1}{3}u = \frac{1}{3}f(3x)\Big|_0^2 = \frac{1}{3}\left[f(6) - f(0)\right]$
$= \frac{1}{3}\left[0 - 2\right]$

(A) -2 **(B)** $-\frac{2}{3}$ **(C)** 0 **(D)** $\frac{2}{3}$ **(E)** 2

$\boxed{FTC}$

41. Let $g(x) = \int_0^{2x} f(t)\,dt$; then $g'(1)$ $\quad g'(x) = \frac{d}{dx}\int_0^{2x} f(t)\,dt = f(2x)\cdot 2$

$g'(1) = f(2)\cdot 2$
$= 4\cdot 2$
$= 8$

(A) = 3.
(B) = 4.
(C) = 6.
(D) = 8.
(E) does not exist, because *f* is not differentiable at $x = 2$.

42. Let $h(x) = x^2 - f(x)$. Find $\int_0^6 h(x)\,dx$. $= \int_0^6 x^2\,dx - \int_0^6 f(x)\,dx = \frac{x^3}{3}\Big|_0^6 -$ Area graph 0 to 6

(A) 22 **(B)** 38 **(C)** 58 **(D)** 70 **(E)** 74

$= \frac{216}{3} - \left[\frac{1}{2}(2)(2) + (2)(2) + \frac{1}{2}(4)(4)\right]$
$= 72 - [2 + 4 + 8]$
$= 72 - 14$
$= 58$

#43 MVT

43. If $f(x)$ is continuous on the closed interval $[a,b]$, then there exists at least one number c, $a < c < b$, such that $\int_a^b f(x)\,dx$ is equal to 49.

(A) $\frac{f(c)}{b-a}$ (B) $f'(c)(b-a)$ (C) $f(c)(b-a)$

(D) $\frac{f'(c)}{b-a}$ (E) $f(c)[f(b)-f(a)]$

#44 Constant Rule

44. If $f(x)$ is continuous on the closed interval $[a,b]$ and k is a constant, then $\int_a^b kf(x)\,dx$ is equal to 50.

(A) $k(b-a)$ (B) $k[f(b)-f(a)]$ (C) $kF(b-a)$, where $\frac{dF(x)}{dx}=f(x)$

(D) $k\int_a^b f(x)\,dx$ (E) $\left.\frac{[kf(x)]^2}{2}\right]_a^b$

#45 FTC

$\sqrt{t^3+1}\cdot 1$

45. $\frac{d}{dt}\int_0^t \sqrt{x^3+1}\,dx =$

(A) $\sqrt{t^3+1}$ (B) $\frac{\sqrt{t^3+1}}{3t^2}$ (C) $\frac{2}{3}(t^3+1)(\sqrt{t^3+1}-1)$ 51.

(D) $3x^2\sqrt{x^3+1}$ (E) none of these

#46 FTC

$(2-u^2)^3\cdot 1$

46. If $F(u)=\int_1^u (2-x^2)^3\,dx$, then $F'(u)$ is equal to

(A) $-6u(2-u^2)^2$ (B) $\frac{(2-u^2)^4}{4}-\frac{1}{4}$ (C) $(2-u^2)^3-1$

(D) $(2-u^2)^3$ (E) $-2u(2-u^2)^3$

#47 FTC

$\sqrt{\sin x^2}\cdot 2x$

47. $\frac{d}{dx}\int_{\pi/2}^{x^2}\sqrt{\sin t}\,dt =$

(A) $\sqrt{\sin t^2}$ (B) $2x\sqrt{\sin x^2}-1$ (C) $\frac{2}{3}(\sin^{3/2}x^2-1)$

(D) $\sqrt{\sin x^2}-1$ (E) $2x\sqrt{\sin x^2}$

48. If $x=4\cos\theta$ and $y=3\sin\theta$, then $\int_2^4 xy\,dx$ is equivalent to 52.

(A) $48\int_{\pi/3}^0 \sin\theta\cos^2\theta\,d\theta$ (B) $48\int_2^4 \sin^2\theta\cos\theta\,d\theta$

(C) $36\int_2^4 \sin\theta\cos^2\theta\,d\theta$ (D) $-48\int_0^{\pi/3}\sin\theta\cos^2\theta\,d\theta$ 53.

(E) $48\int_0^{\pi/3}\sin^2\theta\cos\theta\,d\theta$

ıber

BC ONLY

49. A curve is defined by the parametric equations $x = 2a \tan \theta$ and $y = 2a \cos^2 \theta$, where $0 \leqq \theta \leqq \pi$. Then the definite integral $\pi \int_0^{2a} y^2\, dx$ is equivalent to

(A) $4\pi a^2 \int_0^{\pi/4} \cos^4 \theta\, d\theta$ (B) $8\pi a^3 \int_{\pi/2}^{\pi} \cos^2 \theta\, d\theta$ (C) $8\pi a^3 \int_0^{\pi/4} \cos^2 \theta\, d\theta$

(D) $8\pi a^3 \int_0^{2a} \cos^2 \theta\, d\theta$ (E) $8\pi a^3 \int_0^{\pi/4} \sin \theta \cos^2 \theta\, d\theta$

50. A curve is given parametrically by $x = 1 - \cos t$ and $y = t - \sin t$, where $0 \leqq t \leqq \pi$. Then $\int_0^{3/2} y\, dx$ is equivalent to

(A) $\int_0^{3/2} \sin t(t - \sin t)\, dt$ (B) $\int_{2\pi/3}^{\pi} \sin t(t - \sin t)\, dt$

(C) $\int_0^{2\pi/3} (t - \sin t)\, dt$ (D) $\int_0^{2\pi/3} \sin t(t - \sin t)\, dt$

(E) $\int_0^{3/2} (t - \sin t)\, dt$

(x)

51. When $\int_0^1 \sqrt{1+x^2}\, dx$ is estimated using $n = 5$ subintervals of equal width, which is (are) true?

I. $L(5) = \left(1 + \sqrt{1+0.2^2} + \sqrt{1+0.4^2} + \sqrt{1+0.6^2} + \sqrt{1+0.8^2}\right)$

II. $M(5) = \left(\sqrt{1+0.1^2} + \sqrt{1+0.3^2} + \sqrt{1+0.5^2} + \sqrt{1+0.7^2} + \sqrt{1+0.9^2}\right) \cdot (0.2)$

III. $T(5) = \frac{0.2}{2}\left(1 + 2\sqrt{1+0.2^2} + 2\sqrt{1+0.4^2} + 2\sqrt{1+0.6^2} + 2\sqrt{1+0.8^2} + \sqrt{2}\right)$

(A) II only
(B) III only
(C) I and II only
(D) I and III only
(E) II and III only

52. Find the value of x at which the function $y = x^2$ reaches its average value on the interval [0,10].

(A) 4.642 (B) 5 (C) 5.313 (D) 5.774 (E) 7.071

53. The average value of $f(x) = \begin{cases} x^3, & x < 2 \\ 4x, & x \geq 2 \end{cases}$ on the interval $0 \leq x \leq 5$ is

(A) 8 (B) 9.2 (C) 16 (D) 23
(E) undefined because f is not differentiable on this interval

Answer Key

1. C	**12.** B	**23.** E	**34.** C	**45.** A
2. B	**13.** E	**24.** C	**35.** C	**46.** D
3. E	**14.** C	**25.** B	**36.** E	**47.** E
4. B	**15.** D	**26.** D	**37.** B	**48.** E
5. D	**16.** A	**27.** D	**38.** B	**49.** C
6. A	**17.** C	**28.** D	**39.** D	**50.** D
7. D	**18.** E	**29.** E	**40.** B	**51.** E
8. A	**19.** A	**30.** D	**41.** D	**52.** D
9. C	**20.** E	**31.** D	**42.** C	**53.** B
10. D	**21.** C	**32.** E	**43.** C	
11. B	**22.** A	**33.** C	**44.** D	

Answers Explained

1. (C) The integral is equal to

$$\left(\frac{1}{3}x^3-\frac{1}{2}x^2-x\right)\Bigg|_{-1}^{1}=-\frac{7}{6}-\frac{1}{6}.$$

2. (B) Rewrite as $\int_1^2\left(1-\frac{1}{3}\cdot\frac{1}{x}\right)dx$. This equals

$$\left(x-\frac{1}{3}\ln x\right)\Bigg|_1^2=2-\frac{1}{3}\ln 2-1.$$

3. (E) Rewrite as

$$-\int_0^3(4-t)^{-1/2}(-1\,dt)=-2\sqrt{4-t}\,\Bigg|_0^3=-2(1-2).$$

4. (B) This integral equals

$$\frac{1}{3}\int_{-1}^{0}(3u+4)^{1/2}\cdot 3\,du=\frac{1}{3}\cdot\frac{2}{3}(3u+4)^{3/2}\Bigg|_{-1}^{0}$$

$$=\frac{2}{9}(4^{3/2}-1^{3/2}).$$

5. (D) $\frac{1}{2}\int_2^3\frac{2\,dy}{2y-3}=\frac{1}{2}\ln(2y-3)\Big|_2^3=\frac{1}{2}(\ln 3-\ln 1)$

6. (A) Rewrite as

$$-\frac{1}{2}\int_0^{\sqrt{3}}(4-x^2)^{-1/2}(-2x\,dx)=-\frac{1}{2}\cdot 2\sqrt{4-x^2}\,\Bigg|_0^{\sqrt{3}}=-(1-2).$$

7. (D) $\frac{1}{2}\int_0^1(2t-1)^3(2dt)=\frac{1}{2}\cdot\frac{(2t-1)^4}{4}\Big|_0^1=\frac{1}{2}\left(\frac{(2\cdot 1-1)^4}{4}-\frac{(2\cdot 0-1)^4}{4}\right)$

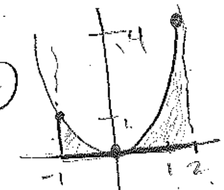
Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

AREA

In Questions 1–11, choose the alternative that gives the area of the region whose boundaries are given.

#1



1. The curve of $y = x^2$, $y = 0$, $x = -1$, and $x = 2$.

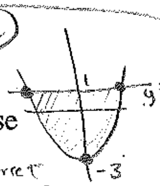
   (A) $\frac{11}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 5 (E) none of these

Top Bottom

$\int_{-1}^{2} (x^2 - 0)\,dx$

$\frac{x^3}{3}\Big|_{-1}^{2} = \frac{8}{3} - \left(-\frac{1}{3}\right) = 3$

#2



2. The parabola $y = x^2 - 3$ and the line $y = 1$.

   (A) $\frac{8}{3}$ (B) 32 (C) $\frac{32}{3}$ (D) $\frac{16}{3}$ (E) none of these

Intersect pt. $x^2 - 3 = 1$ $x^2 = 4$ $x = \pm 2$

Top Bottom

$\int_{-2}^{2} (1 - (x^2 - 3))\,dx$

$\int_{-2}^{2} -x^2 + 4\;dx$

$-\frac{x^3}{3}\Big|_{-2}^{2} + 4x\Big|_{-2}^{2}$

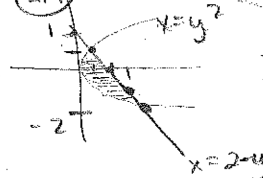
$\left(-\frac{8}{3} + -\frac{8}{3}\right) + (8 - (-8))$

$-\frac{16}{3} + 16 = -\frac{16}{3} + \frac{48}{3}$ $\frac{32}{3}$

3. The curve of $x = y^2 - 1$ and the y-axis.

   (A) $\frac{4}{3}$ (B) $\frac{2}{3}$ (C) $\frac{32}{3}$ (D) $\frac{1}{2}$ (E) none of these

#4



4. The parabola $y^2 = x$ and the line $x + y = 2$.

   (A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{11}{6}$ (D) $\frac{9}{2}$ (E) $\frac{29}{6}$

Intersection pts

$y^2 = 2 - y$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2$ $y = 1$

$\int_{-2}^{1} \left[(2-y) - y^2\right] dy$ right − left

$2y\Big|_{-2}^{1} - \frac{y^2}{2}\Big|_{-2}^{1} - \frac{y^3}{3}\Big|_{-2}^{1}$

$(2-(-4)) - \left(\frac{1}{2} - 2\right) - \left(\frac{1}{3} - \frac{-8}{3}\right)$

$6 + \frac{3}{2} - 3 = \frac{9}{2}$

5. The curve of $y = \frac{4}{x^2 + 4}$, the x-axis, and the vertical lines $x = -2$ and $x = 2$.

   (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 2π (D) π (E) none of these

6. The parabolas $x = y^2 - 5y$ and $x = 3y - y^2$.

   (A) $\frac{32}{3}$ (B) $\frac{139}{6}$ (C) $\frac{64}{3}$ (D) $\frac{128}{3}$ (E) none of these

#6 Intersection pt.

$y^2 - 5y = 3y - y^2$

$2y^2 - 8y = 0$

$2y(y-4) = 0$

$y = 0$ $y = 4$

7. The curve of $y = \frac{2}{x}$ and $x + y = 3$.

   (A) $\frac{1}{2} - 2\ln 2$ (B) $\frac{3}{2}$ (C) $\frac{1}{2} - \ln 4$

   (D) $\frac{5}{2}$ (E) $\frac{3}{2} - \ln 4$

Guess Left/Right if neg → guessed wrong

$\int_0^4 (y^2 - 5y) - (3y - y^2)\,dy$

$\int_0^4 2y^2 - 8y\;dy$

$\frac{2y^3}{3}\Big|_0^4 - \frac{8y^2}{2}\Big|_0^4$

$\frac{2(64)}{3} - 4(16)$

$\frac{128}{3} - \frac{192}{3} = (-)\frac{64}{3}$

picked wrong

8. In the first quadrant, bounded below by the x-axis and above by the curves of $y = \sin x$ and $y = \cos x$.

   (A) $2 - \sqrt{2}$ (B) $2 + \sqrt{2}$ (C) 2 (D) $\sqrt{2}$ (E) $2\sqrt{2}$

#9

$\frac{5\pi}{4}$ Top − Bottom

$\int_{\pi/4}^{5\pi/4} [\sin x - \cos x]\,dx$

$-\cos x \Big|_{\pi/4}^{5\pi/4} - \sin x \Big|_{\pi/4}^{5\pi/4}$

$-\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$

$-\left(-\frac{2\sqrt{2}}{2}\right) - \left(-\frac{2\sqrt{2}}{2}\right)$

$= \sqrt{2} + \sqrt{2}$

9. Bounded above by the curve $y = \sin x$ and below by $y = \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{5\pi}{4}$.

(A) $2\sqrt{2}$ (B) $\frac{2}{\sqrt{2}}$ (C) $\frac{1}{2\sqrt{2}}$

(D) $2(\sqrt{2}-1)$ (E) $2(\sqrt{2}+1)$

10. The curve $y = \cot x$, the line $x = \frac{\pi}{4}$, and the x-axis.

(A) $\ln 2$ (B) $\frac{1}{2}\ln\frac{1}{2}$ (C) 1 (D) $\frac{1}{2}\ln 2$ (E) 2

#11 Intersection

$x^3 - 2x^2 - 3x = 0$

$x(x^2 - 2x - 3) = 0$

$x(x-3)(x+1) = 0$

$x = 0,\ x = 3,\ x = -1$

11. The curve of $y = x^3 - 2x^2 - 3x$ and the x-axis.

(A) $\frac{28}{3}$ (B) $\frac{79}{6}$ (C) $\frac{45}{4}$ (D) $\frac{71}{6}$ (E) none of these

Top − Bottom

$\int_{-1}^{0} [0 - (x^3 - 2x^2 - 3x)]\,dx + \int_0^3 [x^3 - 2x^2 - 3x - 0]\,dx$

$-\frac{x^4}{4}\Big|_{-1}^{0} + \frac{2x^3}{3}\Big|_{-1}^{0} + \frac{3x^2}{2}\Big|_{-1}^{0} + \frac{x^4}{4}\Big|_0^3 - \frac{2x^3}{3}\Big|_0^3 - \frac{3x^2}{2}\Big|_0^3$

$\left(0 + \frac{1}{4}\right) + \left(0 + \frac{2}{3}\right) + \left(0 - \frac{3}{2}\right) + \left(\frac{81}{4}\right) - \left(\frac{54}{3}\right) - \left(\frac{27}{2}\right)$

$\left(\frac{1}{4} + \frac{2}{3} - \frac{3}{2}\right) + \left(\frac{81}{4} - 18 - \frac{27}{2}\right)$

$\left(\frac{3}{12} + \frac{8}{12} - \frac{18}{12}\right) + \left(\frac{81}{4} - \frac{72}{4} - \frac{54}{4}\right)$

$-\frac{7}{12} + \frac{45}{4} = -\frac{7}{12} + \frac{135}{12} = \frac{128}{12}$

12. The total area bounded by the cubic $x = y^3 - y$ and the line $x = 3y$ is equal to

(A) 4 (B) $\frac{16}{3}$ (C) 8 (D) $\frac{32}{3}$ (E) 16

13. The area bounded by $y = e^x$, $y = 2$, and the y-axis is equal to

(A) $3 - e$ (B) $e^2 - 1$ (C) $e^2 + 1$

(D) $2\ln 2 - 1$ (E) $2\ln 2 - 3$

BC ONLY

14. The area enclosed by the ellipse with parametric equations $x = 2\cos\theta$ and $y = 3\sin\theta$ equals

(A) 6π (B) $\frac{9}{2}\pi$ (C) 3π (D) $\frac{3}{2}\pi$ (E) none of these

15. The area enclosed by one arch of the cycloid with parametric equations $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$ equals

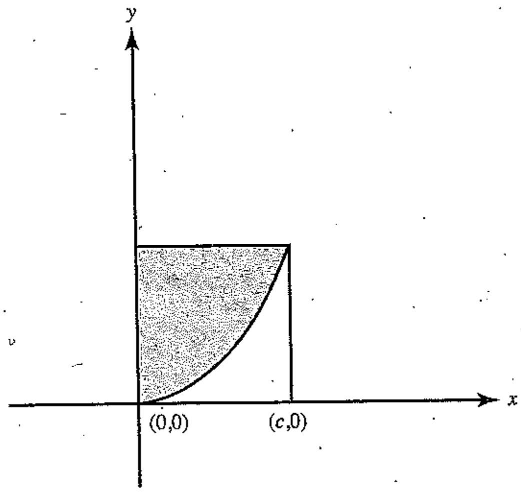
(A) $\frac{3\pi}{2}$ (B) 3π (C) 2π (D) 6π (E) none of these

16. The area enclosed by the curve $y^2 = x(1-x)$ is given by

(A) $2\int_0^1 x\sqrt{1-x}\,dx$ (B) $2\int_0^1 \sqrt{x-x^2}\,dx$ (C) $4\int_0^1 \sqrt{x-x^2}\,dx$

(D) π (E) 2π

17. The figure below shows part of the curve of $y = x^3$ and a rectangle with two vertices at $(0, 0)$ and $(c, 0)$. What is the ratio of the area of the rectangle to the shaded part of it above the cubic?



(A) 3:4 **(B)** 5:4 **(C)** 4:3 **(D)** 3:1 **(E)** 2:1

$\pi \int [r(x)]^2 dx$

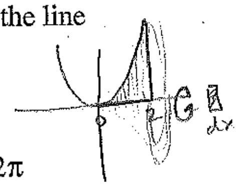
VOLUME

In Questions 18–24 the region whose boundaries are given is rotated about the line indicated. Choose the alternative that gives the volume of the solid generated.

Disk (Not washer)
Top - Bottom^2

18. $y = x^2$, $x = 2$, and $y = 0$; about the x-axis.

(A) $\frac{64\pi}{3}$ **(B)** 8π **(C)** $\frac{8\pi}{3}$ **(D)** $\frac{128\pi}{5}$ **(E)** $\frac{32\pi}{5}$



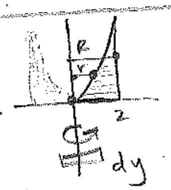
$\pi\int_0^2 (x^2-0)^2 dx$

$\pi\int_0^2 x^4 dx$

$\pi \frac{x^5}{5}\Big|_0^2 = \frac{\pi \cdot 32}{5} - 0$

19. $y = x^2$, $x = 2$, and $y = 0$; about the y-axis.

(A) $\frac{16\pi}{3}$ **(B)** 4π **(C)** $\frac{32\pi}{5}$ **(D)** 8π **(E)** $\frac{8\pi}{3}$



Washer (hole)

intersection $\sqrt{y} = 2$ $(y = 4)$

$R \to x = 2$

$r \to x = \sqrt{y}$

$\pi \int_0^4 [R^2 - r^2] dy$

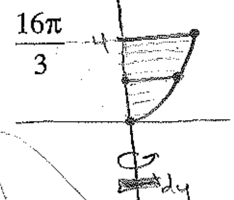
$\pi \int_0^4 [2^2 - y] dy$

$4\pi y\Big|_0^4 - \frac{\pi y^2}{2}\Big|_0^4$

$16\pi - 8\pi$

20. The first quadrant region bounded by $y = x^2$, the y-axis, and $y = 4$; about the y-axis.

(A) 8π **(B)** 4π **(C)** $\frac{64\pi}{3}$ **(D)** $\frac{32\pi}{3}$ **(E)** $\frac{16\pi}{3}$



Disk $= \pi\int_0^4 (\sqrt{y})^2 dy$

$= \pi \int_0^4 y \, dy$

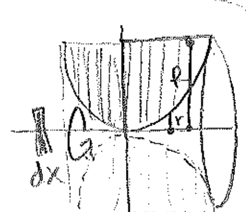
$= \frac{\pi y^2}{2}\Big|_0^4$

$= 8\pi - 0$

21. $y = x^2$ and $y = 4$; about the x-axis.

(A) $\frac{64\pi}{5}$ **(B)** $\frac{512\pi}{15}$ **(C)** $\frac{256\pi}{5}$

(D) $\frac{128\pi}{5}$ **(E)** none of these



Washer (hole)

$R \to y = 4$

$r \to y = x^2$

Intersection

$x^2 = 4$

$x = \pm 2$

$= \pi \int_{-2}^2 [(4)^2 - (x^2)^2] dx$

$= \pi \int_{-2}^2 (16 - x^4) dx$

$= \pi 16x\Big|_{-2}^2 - \frac{\pi x^5}{5}\Big|_{-2}^2$

$= (32\pi + 32\pi) - \left(\frac{32\pi}{5} + \frac{32\pi}{5}\right) = \frac{320\pi}{5} - \frac{64\pi}{5}$

#22 G $y=4$ dy

Disk (no hole)

$\pi\int_{-2}^{2}(4-x^2)^2$ top − Bottom $\to (4-x^2)(4-x^2)$

$\pi\int_{-2}^{2}(16-2x^2+x^4)\,dx = \pi 16x\Big|_{-2}^{2} - 2\pi\frac{x^3}{3}\Big|_{-2}^{2} + \frac{\pi x^5}{5}\Big|_{-2}^{2} = (32\pi+32\pi)-\left(\frac{16\pi}{3}+\frac{16\pi}{3}\right)+\left(\frac{32\pi}{5}+\frac{32\pi}{5}\right)$

$= 64\pi - \frac{32\pi}{3} + \frac{64\pi}{5}$

$= \frac{512\pi}{15}$

22. $y = x^2$ and $y = 4$; about the line $y = 4$.

(A) $\frac{256\pi}{15}$ (B) $\frac{256\pi}{5}$ (C) $\frac{512\pi}{5}$ (D) $\frac{512\pi}{15}$ (E) $\frac{64\pi}{3}$

23. An arch of $y = \sin x$ and the x-axis; about the x-axis.

(A) $\frac{\pi}{2}\left(\pi - \frac{1}{2}\right)$ (B) $\frac{\pi^2}{2}$ (C) $\frac{\pi^2}{4}$ (D) π^2 (E) $\pi(\pi - 1)$

Disk (hole) top − bott

$=\pi\int_0^{\pi}(\sin x - 0)^2\,dx$

$\pi G\ dx$ $\pi\int_0^{\pi}\sin^2 x\,dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$ Trig identity

$\pi\int_0^{\pi}\left(\frac{1-\cos 2x}{2}\right)dx = \frac{\pi}{2}\int_0^{\pi}[1-\cos 2x]\,dx = \frac{\pi}{4}\int_0^{\pi}[1-\cos u]\,du = \frac{\pi}{2}\left[x-\frac{1}{2}\sin 2x\right]_0^{\pi} = \frac{\pi}{2}[\ldots]$

$u = 2x$, $du = 2\,dx$

24. A trapezoid with vertices at (2, 0), (2, 2), (4, 0), and (4, 4); about the x-axis.

(A) $\frac{56\pi}{3}$ (B) $\frac{128\pi}{3}$ (C) $\frac{92\pi}{3}$

(D) $\frac{112\pi}{3}$ (E) none of these

25. The base of a solid is a circle of radius a, and every plane section perpendicular to a diameter is a square. The solid has volume

(A) $\frac{8}{3}a^3$ (B) $2\pi a^3$ (C) $4\pi a^3$ (D) $\frac{16}{3}a^3$ (E) $\frac{8\pi}{3}a^3$

$V = (2y)^2\,dx$ $\quad 1\int_{-a}^{a}4(a^2-x^2)\,dx \to \frac{16}{3}a^3$

$= 4y^2\,dx$

$= 4(a^2-x^2)$

26. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is

(A) $\frac{64\sqrt{3}}{3}$ (B) $64\sqrt{3}$ (C) $32\sqrt{3}$

(D) 32 (E) none of these

$1\int_0^1[e^{-x}-0]^2\,dx$

$1\int_0^1 e^{-2x}\,dx$ $\quad u=-2x$, $du=-2dx$

27. The base of a solid is the region bounded by $y = e^{-x}$, the x-axis, the y-axis, and the line $x = 1$. Each cross section perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{e^2}{2}$ (B) $e^2 - 1$ (C) $1 - \frac{1}{e^2}$

(D) $\frac{e^2 - 1}{2}$ (E) $\frac{1}{2}\left(1 - \frac{1}{e^2}\right)$

$-\frac{1}{2}e^{-2x}\Big|_0^1$

$-\frac{1}{2}\left[e^{-2}-e^{0}\right]$

$-\frac{1}{2}\left[\frac{1}{e^2}-1\right] = \frac{1}{2}\left(1-\frac{1}{e^2}\right)$

BC ONLY

ARC LENGTH

28. The length of the arc of the curve $y^2 = x^3$ cut off by the line $x = 4$ is

(A) $\frac{4}{3}(10\sqrt{10} - 1)$ (B) $\frac{8}{27}(10^{3/2} - 1)$ (C) $\frac{16}{27}(10^{3/2} - 1)$

(D) $\frac{16}{27}10\sqrt{10}$ (E) none of these

BC ONLY

29. The length of the arc of $y = \ln \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{3}$ equals

(A) $\ln \frac{\sqrt{3}+2}{\sqrt{2}+1}$ (B) 2 (C) $\ln(1+\sqrt{3}-\sqrt{2})$

(D) $\sqrt{3}-2$ (E) $\frac{\ln(\sqrt{3}+2)}{\ln(\sqrt{2}+1)}$

IMPROPER INTEGRALS

30. $\int_0^\infty e^{-x}\,dx =$

(A) 1 (B) $\frac{1}{e}$ (C) -1 (D) $-\frac{1}{e}$ (E) none of these

31. $\int_0^e \frac{du}{u} =$

(A) 1 (B) $\frac{1}{e}$ (C) $-\frac{1}{e^2}$ (D) -1 (E) none of these

32. $\int_1^2 \frac{dt}{\sqrt[3]{t-1}} =$

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 1 (E) none of these

33. $\int_2^4 \frac{dx}{(x-3)^{2/3}} =$

(A) 6 (B) $\frac{6}{5}$ (C) $\frac{2}{3}$ (D) 0 (E) none of these

34. $\int_2^4 \frac{dx}{(x-3)^2} =$

(A) 2 (B) -2 (C) 0 (D) $\frac{2}{3}$ (E) none of these

35. $\int_0^{\pi/2} \frac{\sin x}{\sqrt{1-\cos x}}\,dx$

(A) -2 (B) $\frac{2}{3}$ (C) 2 (D) $\frac{1}{2}$ (E) none of these

BC ONLY

In Questions 36–40, choose the alternative that gives the area, if it exists, of the region described.

36. In the first quadrant under the curve of $y = e^{-x}$.

(A) 1 (B) e (C) $\frac{1}{e}$ (D) 2 (E) none of these

37. In the first quadrant under the curve of $y = xe^{-x^2}$.

(A) 2 (B) $\frac{2}{e}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2e}$ (E) none of these

38. In the first quadrant above $y = 1$, between the y-axis and the curve $xy = 1$.

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 4 (E) none of these

39. Between the curve $y = \frac{4}{1+x^2}$ and the x-axis.

(A) 2π (B) 4π (C) 8π (D) π (E) none of these

40. Above the x-axis, between the curve $y = \frac{4}{\sqrt{1-x^2}}$ and its asymptotes.

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π (E) none of these

In Questions 41 and 42, choose the alternative that gives the volume, if it exists, of the solid generated.

41. $y = \frac{1}{x}$, at the left by $x = 1$, and below by $y = 0$; about the x-axis.

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π (E) none of these

42. The first-quadrant region under $y = e^{-x}$; about the x-axis.

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) 4π (E) none of these

Part B. Directions: Some of the following questions require the use of a graphing calculator.

AREA

In Questions 43–47, choose the alternative that gives the area of the region whose boundaries are given.

intersection
$2-x^2=x-4$
$0=x^2+x-6$
$(x+3)(x-2)$
$x=-3,\ x=2$

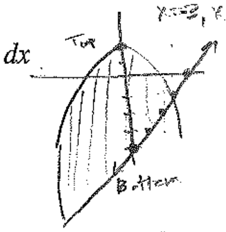
$\int_{-3}^{2}$ top − Bottom $[2-x^2]-[x-4]\,dx$

up 2 open down ... m=1 b=−4

43. The area bounded by the parabola $y = 2 - x^2$ and the line $y = x - 4$ is given by

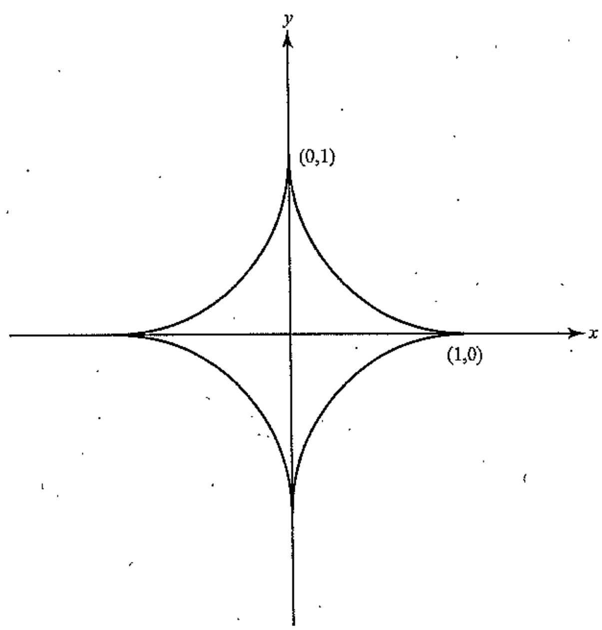
(A) $\int_{-2}^{3} (6 - x - x^2)\, dx$ (B) $\int_{-2}^{1} (2 + x + x^2)\, dx$ (C) $\int_{-3}^{2} (6 - x - x^2)\, dx$

(D) $2\int_{0}^{\sqrt{2}} (2 - x^2)\, dx + \int_{-3}^{2} (4 - x)\, dx$ (E) none of these



$\int_{-3}^{2} (6-x-x^2)\,dx$

BC ONLY



44. The area enclosed by the hypocycloid with parametric equations $x = \cos^3 t$ and $y = \sin^3 t$ as shown in the above diagram is

(A) $3\int_{\pi/2}^{0} \sin^4 t \cos^2 t\, dt$ (B) $4\int_{0}^{1} \sin^3 t\, dt$ (C) $-4\int_{\pi/2}^{0} \sin^6 t\, dt$

(D) $12\int_{0}^{\pi/2} \sin^4 t \cos^2 t\, dt$ (E) none of these

45. Suppose the following is a table of coordinates for $y = f(x)$, given that f is continuous on [1, 5]: Area Trap $= \frac{1}{2}\cdot b(h_1+h_2)$

b→ 1 1 1 1

x	1	2	3	4	5
y	1.62	4.15	7.5	9.0	12.13

$T(4) = \frac{1}{2}\left[1(1.62+4.15)+1(4.15+7.5)+1(7.5+9)+1(9+12.13)\right] =$

If a trapezoid sum in used, with $n = 4$, then the area under the curve, from $x = 1$ to $x = 5$, is equal, to two decimal places, to

(A) 6.88 (B) 13.76 (C) 20.30 (D) 25.73 (E) 27.53

BC ONLY

46. The area A enclosed by the four-leaved rose $r = \cos 2\theta$ equals, to three decimal places,

(A) 0.785 (B) 1.571 (C) 2.071 (D) 3.142 (E) 6.283

47. The area bounded by the small loop of the limaçon $r = 1 - 2\sin\theta$ is given by the definite integral

(A) $\displaystyle\int_{\pi/3}^{5\pi/3} \left[\frac{1}{2}(1-2\sin\theta)\right]^2 d\theta$

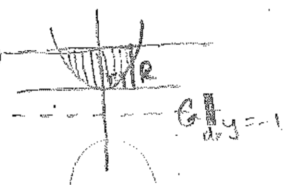
(B) $\displaystyle\int_{7\pi/6}^{3\pi/2} (1-2\sin\theta)^2\, d\theta$

(C) $\displaystyle\int_{\pi/6}^{\pi/2} (1-2\sin\theta)^2\, d\theta$

(D) $\displaystyle\int_{0}^{\pi/6} \left[\frac{1}{2}(1-2\sin\theta)\right]^2 d\theta + \int_{5\pi/6}^{\pi} \left[\frac{1}{2}(1-2\sin\theta)\right]^2 d\theta$

(E) $\displaystyle\int_{0}^{\pi/3} (1-2\sin\theta)^2\, d\theta$

#48



VOLUME

In Questions 48–54 the region whose boundaries are given is rotated about the line indicated. Choose the alternative that gives the volume of the solid generated.

Washer [hole]
$R \to y = 4$
$r \to y = x^2$

Intersection
$x^2 = 4$
$x = \pm 2$

$\pi \int ((4)^2 - (x^2)^2)\,dx$

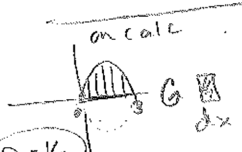
$\pi \int^2 16 - x^4\, dx =$

48. $y = x^2$ and $y = 4$; about the line $y = -1$.

(A) $4\pi\displaystyle\int_{-1}^{4} (y+1)\sqrt{y}\, dy$ (B) $2\pi\displaystyle\int_{0}^{2} (4-x^2)^2\, dx$ (C) $\pi\displaystyle\int_{-2}^{2} (16-x^4)\, dx$

(D) $2\pi\displaystyle\int_{0}^{2} (24-2x^2-x^4)\, dx$ (E) none of these

on calc



Disk

Intersection
$3x - x^2 = 0$
$x(3-x) = 0$
$x = 0,\ x = 3$

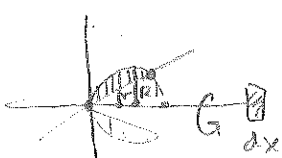
Top - Bottom
$\pi \int_0^3 [(3x-x^2)^2 - 0^2]\,dx$

49. $y = 3x - x^2$ and $y = 0$; about the x-axis.

(A) $\pi\displaystyle\int_{0}^{3} (9x^2+x^4)\, dx$ (B) $\pi\displaystyle\int_{0}^{3} (3x-x^2)^2\, dx$ (C) $\pi\displaystyle\int_{0}^{\sqrt{3}} (3x-x^2)\, dx$

(D) $2\pi\displaystyle\int_{0}^{3} y\sqrt{9-4y}\, dy$ (E) $\pi\displaystyle\int_{0}^{9/4} y^2\, dy$

50. $y = 3x - x^2$ and $y = x$; about the x-axis.



Intersection
$3x - x^2 = x$
$2x - x^2 = 0$
$x(2-x) = 0$
$x = 0,\ x = 2$

(A) $\pi\displaystyle\int_{0}^{3/2} [(3x-x^2)^2 - x^2]\, dx$ (B) $\pi\displaystyle\int_{0}^{2} (9x^2-6x^3)\, dx$

(C) $\pi\displaystyle\int_{0}^{2} [(3x-x^2)^2 - x^2]\, dx$ (D) $\pi\displaystyle\int_{0}^{3} [(3x-x^2)^2 - x^4]\, dx$

(E) $\pi\displaystyle\int_{0}^{3} (2x-x^2)^2\, dx$

Washer (hole)

$\pi \int_0^2 (R^2 - r^2)\,dx$

$\pi \int_0^2 [(3x-x^2)^2 - (x)^2]\,dx$

$x = e^y \quad x = e$ dy   Intersection $e^y = e$ $(y=1)$ $\boxed{\text{DISK}}$ $\pi \int^1 [e - e^y]^2\, dy$ Right − Left   y values 0 b/c dy

51. $y = \ln x, y = 0, x = e$; about the line $x = e$.

(A) $\pi \int_1^e (e - x) \ln x \, dx$ **(B)** $\pi \int_0^1 (e - e^y)^2 \, dy$ **(C)** $2\pi \int_1^e (e - \ln x) \, dx$

(D) $\pi \int_0^e (e^2 - 2e^{y+1} + e^{2y}) \, dy$ **(E)** none of these

52. The curve with parametric equations $x = \tan\theta$, $y = \cos^2\theta$, and the lines $x = 0$, $x = 1$, and $y = 0$; about the x-axis.

BC ONLY

(A) $\pi \int_0^{\pi/4} \cos^4\theta \, d\theta$ **(B)** $\pi \int_0^{\pi/4} \cos^2\theta \sin\theta \, d\theta$ **(C)** $\pi \int_0^{\pi/4} \cos^2\theta \, d\theta$

(D) $\pi \int_0^1 \cos^2\theta \, d\theta$ **(E)** $\pi \int_0^1 \cos^4\theta \, d\theta$

53. A sphere of radius r is divided into two parts by a plane at distance h $(0 < h < r)$ from the center. The volume of the smaller part equals

CHALLENGE

(A) $\frac{\pi}{3}(2r^3 + h^3 - 3r^2h)$ **(B)** $\frac{\pi h}{3}(3r^2 - h^2)$ **(C)** $\frac{4}{3}\pi r^3 + \frac{h^3}{3} - r^2h$

(D) $\frac{\pi}{3}(2r^3 + 3r^2h - h^3)$ **(E)** none of these

54. If the curves of $f(x)$ and $g(x)$ intersect for $x = a$ and $x = b$ and if $f(x) > g(x) > 0$ for all x on (a, b), then the volume obtained when the region bounded by the curves is rotated about the x-axis is equal to

(A) $\pi \int_a^b f^2(x)\, dx - \int_a^b g^2(x)\, dx$

(B) $\pi \int_a^b [f(x) - g(x)]^2 \, dx$

(C) $2\pi \int_a^b x[f(x) - g(x)] \, dx$

(D) $\pi \int_a^b [f^2(x) - g^2(x)] \, dx$

(E) none of these

ARC LENGTH

BC ONLY

55. The length of one arch of the cycloid $\begin{matrix} x = t - \sin t \\ y = 1 - \cos t \end{matrix}$ equals

(A) $\int_0^{\pi} \sqrt{1 - \cos t}\, dt$ **(B)** $\int_0^{2\pi} \sqrt{\frac{1 - \cos t}{2}}\, dt$ **(C)** $\int_0^{\pi} \sqrt{2 - 2\cos t}\, dt$

(D) $\int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$ **(E)** $2\int_0^{\pi} \sqrt{\frac{1 - \cos t}{2}}\, dt$

BC ONLY

56. The length of the arc of the parabola $4x = y^2$ cut off by the line $x = 2$ is given by the integral

(A) $\int_{-1}^{1} \sqrt{x^2+1}\,dx$ (B) $\frac{1}{2}\int_{0}^{2} \sqrt{4+y^2}\,dy$ (C) $\int_{-1}^{1} \sqrt{1+x}\,dx$

(D) $\int_{0}^{2\sqrt{2}} \sqrt{4+y^2}\,dy$ (E) none of these

CHALLENGE

57. The length of $x = e^t \cos t$, $y = e^t \sin t$ from $t = 2$ to $t = 3$ is equal to

(A) $\sqrt{2}e^2\sqrt{e^2-1}$ (B) $\sqrt{2}(e^3-e^2)$ (C) $2(e^3-e^2)$

(D) $e^3(\cos 3+\sin 3)-e^2(\cos 2+\sin 2)$ (E) none of these

IMPROPER INTEGRALS

58. Which one of the following is an improper integral?

(A) $\int_{0}^{2} \frac{dx}{\sqrt{x+1}}$ (B) $\int_{-1}^{1} \frac{dx}{1+x^2}$ (C) $\int_{0}^{2} \frac{x\,dx}{1-x^2}$

(D) $\int_{0}^{\pi/3} \frac{\sin x\,dx}{\cos^2 x}$ (E) none of these

59. Which one of the following improper integrals diverges?

(A) $\int_{1}^{\infty} \frac{dx}{x^2}$ (B) $\int_{0}^{\infty} \frac{dx}{e^x}$ (C) $\int_{-1}^{1} \frac{dx}{\sqrt[3]{x}}$

(D) $\int_{-1}^{1} \frac{dx}{x^2}$ (E) none of these

60. Which one of the following improper integrals diverges?

(A) $\int_{0}^{\infty} \frac{dx}{1+x^2}$ (B) $\int_{0}^{1} \frac{dx}{x^{1/3}}$ (C) $\int_{0}^{\infty} \frac{dx}{x^3+1}$

(D) $\int_{0}^{\infty} \frac{dx}{e^x+2}$ (E) $\int_{1}^{\infty} \frac{dx}{x^{1/3}}$

Answer Key

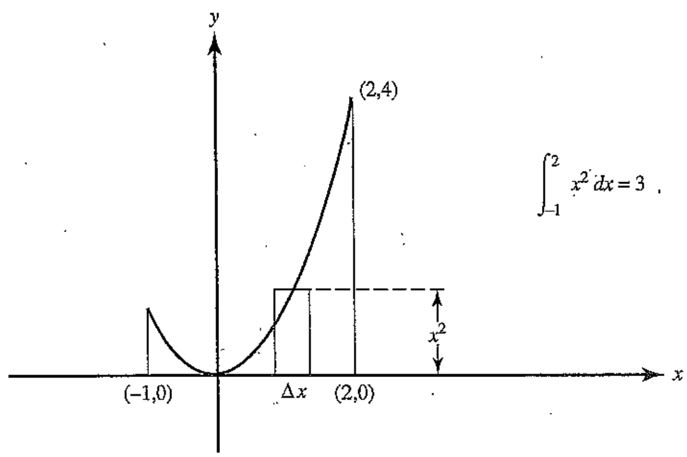
1. C	**14.** A	**27.** E	**40.** D	**53.** A
2. C	**15.** B	**28.** C	**41.** B	**54.** D
3. A	**16.** B	**29.** A	**42.** A	**55.** D
4. D	**17.** C	**30.** A	**43.** C	**56.** D
5. D	**18.** E	**31.** E	**44.** D	**57.** B
6. C	**19.** D	**32.** B	**45.** E	**58.** C
7. E	**20.** A	**33.** A	**46.** B	**59.** D
8. A	**21.** C	**34.** E	**47.** C	**60.** E
9. A	**22.** D	**35.** C	**48.** D	
10. D	**23.** B	**36.** A	**49.** B	
11. D	**24.** A	**37.** C	**50.** C	
12. C	**25.** D	**38.** E	**51.** B	
13. D	**26.** B	**39.** B	**52.** C	

Answers Explained

AREA

We give below, for each of Questions 1–17, a sketch of the region, and indicate a typical element of area. The area of the region is given by the definite integral. We exploit symmetry wherever possible.

1. (C)



$$\int_{-1}^{2} x^2\,dx = 3\ .$$