AP Calculus Exam Review — Weekly Plans

Monday	Tuesday	Wednesday	Thursday	Friday	Sat/Sun
In-class Mock 2015 Exam: 15 Non-Calc MC 1 Non-Calc FRQ	In-class Review: Non-Calculator Questions (MC and FRQ) Exam Grade	In-class Mock 2015 Exam: 10 Calculator MC 1 Calculator FRQ	In-class Review: Non-Calculator Questions (MC and FRQ) Exam Grade	In Class Mock 2015 Exam: 1 Calculator FRQ 2 Non-Calc FRQ	In-class: We are not here ☺

Weekly Homework Due Friday

1. Week 1: Limits: pg 104 #1, 3, 4, 5, 6, 7, 8, 9, 12, 14, 15, 17, 32, 34, 35, 36, 38, 40, 41, 42 (15/20 problems)

   Derivatives: pg 136 # 1, 3, 5, 6, 7, 8, 9, 11, 16, 17, 18, 19, 21, 23, 24, 27, 29, 35, 36, 37, 38, 39, 40, 41, 44, 49, 57, 58, 59, 60, 62, 65, 67, 73, 74, 77, 103 (25/37 problem)

   App. Deriv: pg 1, 3, 4, 5, 7, 12, 13, 14, 15, 16, 17, 19, 21, 22, 23, 26, 31, 33, 37, 40, 42, 43, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 67, 68, 77, 78, 79, 84, 85, 87, 88, 90, 91 (30/45 problems)

   Complete: 70/102 (Homework Grade)

2. Week 2: Antidifferentiation: pg. 231 # 1, 3, 5, 7, 9, 11, 12, 14, 16, 17, 19, 20, 23, 27, 36, 39, 43, 45, 49, 60, 76, 78 (20/22 problems)

   Definite Integrals: pg. 279 # 1, 2, 3, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 26, 27, 28, 29, 33, 34, 36, 40, 41, 42, 43, 44, 45, 46, 47 (30/ 32 problems)

   App Integration: pg 319 # 1, 2, 3, 4, 6, 7, 9, 11, 13, 18, 19, 20, 21, 22, 23, 25, 26, 43, 45, 48, 49, 50, 51 (20/23 problems)

   Complete: 70/77 (Homework Grade)

3. Week 3: Particle Motion: pg 358 #1, 2, 3, 4, 5, 6, 7, 14, 15, 16, 17, 18, 19, 20, 23, 24, 26, 27, 28, 30 (20/20 problems)

   Differential Equations: pg. 393 # 1, 2, 3, 9, 11, 13, 14, 16, 19, 21, 22, 23, 25, 26, 27, 28, 29, 30, 33, 34, 35, 37, 40, 42, 45, 46, 51, 52 (20/28 problems)

   Complete: 40/48 (HW Grade)

Practice Exam Due Monday (45 MC/6 FRQ)

(Quiz Grade)

Key

Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

Plug in: $\lim_{x\to 2} \frac{0}{8} = 0$

L'Hopital won't work b/c it's not indeterminant 0/0 or ∞/∞

1. $\lim_{x\to 2} \frac{x^2-4}{x^2+4}$ is

(A) 1 (B) 0 (C) $-\frac{1}{2}$ (D) −1 (E) ∞

2. $\lim_{x\to \infty} \frac{4-x^2}{x^2-1}$ is

(A) 1 (B) 0 (C) −4 (D) −1 (E) ∞

#3 Plug in $\frac{0}{0}$ indet.

Method 1 Factor: $\frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+1)} = \frac{1}{x+1} = \frac{1}{4}$

Method 2 L'Hopital: $\frac{1}{2x-2} = \frac{1}{6-2} = \frac{1}{4}$

3. $\lim_{x\to 3} \frac{x-3}{x^2-2x-3}$ is

(A) 0 (B) 1 (C) $\frac{1}{4}$ (D) ∞ (E) none of these

#4 Plug in: $\frac{0}{0}$ indet

Meth 1 Factor: $\frac{x}{x} = 1$ | L'Hopital: $\frac{1}{1} = 1$

4. $\lim_{x\to 0} \frac{x}{x}$ is

(A) 1 (B) 0 (C) ∞ (D) −1 (E) nonexistent

#5 Plug in: $\frac{0}{0}$ indet.

Meth 1 Factor: $\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)} = \frac{12}{4}$ | Meth 2 L'Hop: $\frac{3x^2}{2x} = \frac{3x}{2} = \frac{6}{2} = 3$

5. $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ is

(A) 4 (B) 0 (C) 1 (D) 3 (E) ∞

6. $\lim_{x\to \infty} \frac{4-x^2}{4x^2-x-2}$ is

#6 Plug in: $\frac{\infty}{\infty}$ indet.

Dominance: $\frac{-x^2}{4x^2} = -\frac{1}{4}$ | L'Hopital: $\frac{-2x}{8x} = \frac{-2}{8} = -\frac{1}{4}$

(A) −2 (B) $-\frac{1}{4}$ (C) 1 (D) 2 (E) nonexistent

#7 Plug in: ∞/∞ indet

(dominance): $\frac{5x^3}{20x^2} = \frac{x}{4} = -\infty$ | L'Hop: $\frac{15x^2}{40x} = \frac{30x}{40} = -\infty$

7. $\lim_{x\to -\infty} \frac{5x^3+27}{20x^2+10x+9}$ is

(A) −∞ (B) −1 (C) 0 (D) 3 (E) ∞

#8 Plug in: ∞/∞

(dominance): $\frac{3x^2}{x^3} = \frac{3}{x} = \frac{3}{\infty} = 0$ | L'Hop: $\frac{6x}{3x^2} = \frac{6}{6x} = \frac{1}{x} = \frac{1}{\infty} = 0$

8. $\lim_{x\to \infty} \frac{3x^2+27}{x^3-27}$ is

(A) 3 (B) ∞ (C) 1 (D) −1 (E) 0

Plug in: $\frac{1}{2^\infty} = \frac{1}{\infty} = 0$

9. $\lim_{x\to \infty} \frac{2^{-x}}{2^x}$ ~~is~~ $= \frac{1}{2^{2x}}$

(A) −1 (B) 1 (C) 0 (D) ∞ (E) none of these

10. $\lim_{x\to -\infty} \frac{2^{-x}}{2^x}$ is

(A) −1 (B) 1 (C) 0 (D) ∞ (E) none of these

11. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}$ L'Hopital $\dfrac{\cos 5x \cdot 5}{1}\Big|_{x=0} = \dfrac{5\cos(0)}{1} = 5$

(A) $=0$ **(B)** $=\frac{1}{5}$ **(C)** $=1$ **(D)** $=5$ **(E)** does not exist

12. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{3x} =$ L'Hopital $\dfrac{\cos 2x \cdot 2}{3}\Big|_{x=0} = \dfrac{2\cos 0}{3} = \dfrac{2}{3}$

(A) $=0$ **(B)** $=\frac{2}{3}$ **(C)** $=1$ **(D)** $=\frac{3}{2}$ **(E)** does not exist

13. The graph of $y = \arctan x$ has

(A) vertical asymptotes at $x = 0$ and $x = \pi$
(B) horizontal asymptotes at $y = \pm\frac{\pi}{2}$
(C) horizontal asymptotes at $y = 0$ and $y = \pi$
(D) vertical asymptotes at $x = \pm\frac{\pi}{2}$
(E) none of these

14. The graph of $y = \dfrac{x^2-9}{3x-9}$ has

(A) a vertical asymptote at $x = 3$ **(B)** a horizontal asymptote at $y = \frac{1}{3}$
(C) a removable discontinuity at $x = 3$ **(D)** an infinite discontinuity at $x = 3$
(E) none of these

15. $\lim\limits_{x\to 0}\dfrac{\sin x}{x^2+3x}$ is L'Hopital $\dfrac{\cos x}{2x+3}\Big|_{x=0} = \dfrac{\cos 0}{0+3} = \dfrac{1}{3}$

(A) 1 **(B)** $\frac{1}{3}$ **(C)** 3 **(D)** ∞ **(E)** $\frac{1}{4}$

16. $\lim\limits_{x\to 0}\sin\frac{1}{x}$ is

(A) ∞ **(B)** 1 **(C)** nonexistent **(D)** -1 **(E)** none of these

17. Which statement is true about the curve $y = \dfrac{2x^2+4}{2+7x-4x^2}$?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
(B) The line $x = 1$ is a vertical asymptote.
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
(D) The graph has no vertical or horizontal asymptote.
(E) The line $y = 2$ is a horizontal asymptote.

① $\lim\limits_{x\to\pm\infty} f(x)$ = H.A. = Dominance
$= \dfrac{2x^2}{-4x^2} = -\dfrac{1}{2}$ H.A

② V.A when denom = 0
$-(4x^2-7x-2) = 0$ $ac = -8$, $b = -7$ $(-8, 1)$
$(4x^2-8x)+(1x-2) = 0$
$4x(x-2)+1(x-2) = 0$
$(x-2)(4x+1) = 0$ @ $x = 2$ or $x = -1/4$

18. $\lim\limits_{x\to\infty}\dfrac{2x^2+1}{(2-x)(2+x)}$ is $=$ Dominance $\dfrac{2x^2}{-x^2} = -2.$

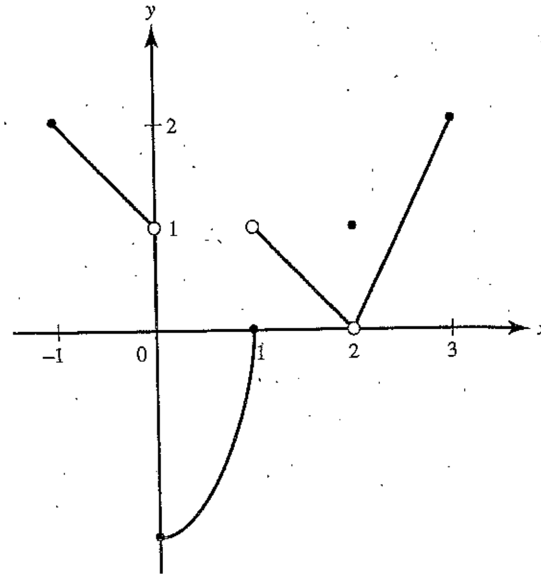
(A) -4 **(B)** -2 **(C)** 1 **(D)** 2 **(E)** nonexistent

☆ you could us L'Hopital but it's harder here b/c product rule

$\dfrac{4x}{(2-x)(1)+(-1)(2+x)} = \dfrac{4x}{2-x-2-x} = \dfrac{4x}{-2x} = \dfrac{4}{-2} = -2$

Questions 32–36 are based on the function f shown in the graph and defined below:

$$f(x)=\begin{cases}1-x & (-1\le x<0)\\ 2x^2-2 & (0\le x\le 1)\\ -x+2 & (1<x<2)\\ 1 & (x=2)\\ 2x-4 & (2<x\le 3)\end{cases}$$



32. $\lim\limits_{x\to 2} f(x)$ y = 0 as x approaches 2

(A) equals 0   (B) equals 1   (C) equals 2
(D) does not exist   (E) none of these

33. The function f is defined on [−1,3]

(A) if $x \neq 0$ (B) if $x \neq 1$ (C) if $x \neq 2$
(D) if $x \neq 3$ (E) at each x in [−1,3]

34. The function f has a removable discontinuity at

(A) $x=0$ (B) $x=1$ (C) $x=2$ (D) $x=3$ (E) none of these

35. On which of the following intervals is f continuous?

(A) $-1 \le x \le 0$ (B) $0<x<1$ ✓   (C) $1 \le x \le 2$
(D) $2 \le x \le 3$ (E) none of these

36. The function f has a jump discontinuity at

(A) $x=-1$ (B) $x=1$ (C) $x=2$
(D) $x=3$ (E) none of these

CHALLENGE

37. $\lim\limits_{x\to 0}\sqrt{3+\arctan\frac{1}{x}}$ is

(A) $-\infty$ (B) $\sqrt{3-\frac{\pi}{2}}$ (C) $\sqrt{3+\frac{\pi}{2}}$
(D) ∞ (E) none of these

38. Suppose $\lim_{x \to -3^-} f(x) = -1$, $\lim_{x \to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements is (are) true?

I. $\lim_{x \to -3} f(x) = -1$. ✓

II. f is continuous everywhere except at $x = -3$. Not 100% sure of this. We just looked @ 1 pt.

III. f has a removable discontinuity at $x = -3$. ✓

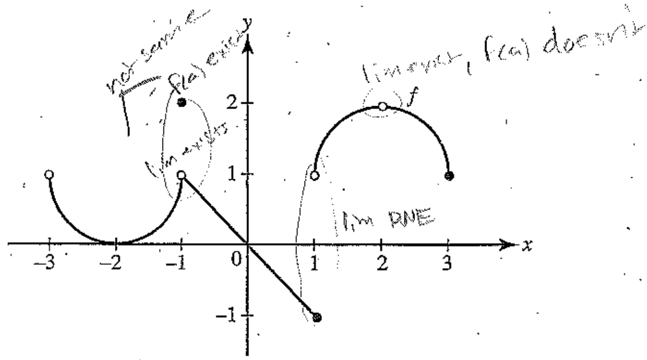
(A) None of them (B) I only (C) III only
(D) I and III only (E) All of them

39. If $y = \dfrac{1}{2 + 10^{\frac{1}{x}}}$, then $\lim_{x \to 0} y$ is

CHALLENGE

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) nonexistent

Questions 40–42 are based on the function f shown in the graph.



40. For what value(s) of a is it true that $\lim_{x \to a} f(x)$ exists and $f(a)$ exists, but $\lim_{x \to a} f(x) \neq f(a)$? It is possible that $a =$

(A) −1 only (B) 1 only (C) 2 only
(D) −1 or 1 only (E) −1 or 2 only

41. $\lim_{x \to a} f(x)$ does not exist for $a =$

(A) ~~−1~~ only (B) 1 only (C) ~~2~~ only
(D) 1 and ~~2~~ only (E) −1, 1, and ~~2~~

42. Which statements about limits at $x = 1$ are true?

I. $\lim_{x \to 1^-} f(x)$ exists. ✓

II. $\lim_{x \to 1^+} f(x)$ exists. ✓

III. $\lim_{x \to 1} f(x)$ exists ✗ Not same!!!

(A) none of I, II, or III (B) I only (C) II only
(D) I and II only (E) I, II, and III

Answer Key

1. B	**12.** B	**23.** B	**34.** C
2. D	**13.** B	**24.** B	**35.** B
3. C	**14.** C	**25.** C	**36.** B
4. A	**15.** B	**26.** C	**37.** E
5. D	**16.** C	**27.** D	**38.** D
6. B	**17.** A	**28.** D	**39.** E
7. A	**18.** B	**29.** E	**40.** A
8. E	**19.** B	**30.** E	**41.** B
9. C	**20.** E	**31.** A	**42.** D
10. D	**21.** A	**32.** A	
11. D	**22.** C	**33.** E	

Answers Explained

1. (B) The limit as $x \to 2$ is $0 \div 8$.

2. (D) Use the Rational Function Theorem (page 98). The degrees of $P(x)$ and $Q(x)$ are the same.

3. (C) Remove the common factor $x - 3$ from numerator and denominator.

4. (A) The fraction equals 1 for all nonzero x.

5. (D) Note that $\frac{x^3-8}{x^2-4}=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$.

6. (B) Use the Rational Function Theorem.

7. (A) Use the Rational Function Theorem.

8. (E) Use the Rational Function Theorem.

9. (C) The fraction is equivalent to $\frac{1}{2^{2x}}$; the denominator approaches ∞.

10. (D) Since $\frac{2^{-x}}{2^x}=2^{-2x}$, therefore, as $x \to -\infty$, the fraction $\to +\infty$.

11. (D) $\lim_{x\to 0}\frac{\sin 5x}{x}=\lim_{x\to 0}\frac{\sin 5x}{x}\cdot\frac{5}{5}=5\lim_{x\to 0}\frac{\sin 5x}{5x}=5$

12. (B) $\lim_{x\to 0}\frac{\sin 2x}{3x}=\frac{1}{3}\lim_{x\to 0}\frac{\sin 2x}{x}\cdot\frac{2}{2}=\frac{2}{3}\lim_{x\to 0}\frac{\sin 2x}{2x}=\frac{2}{3}$

13. (B) Because the graph of $y=\tan x$ has vertical asymptotes at $x=\pm\frac{\pi}{2}$, the graph of the inverse function $y=\arctan x$ has horizontal asymptotes at $y=\pm\frac{\pi}{2}$.

14. (C) Since $\frac{x^2-9}{3x-9}=\frac{(x-3)(x+3)}{3(x-3)}=\frac{x+3}{3}$ (provided $x\neq 3$), y can be defined to be equal to 2 at $x=3$, removing the discontinuity at that point.

15. (B) Note that $\frac{\sin x}{x^2+3x}=\frac{\sin x}{x(x+3)}=\frac{\sin x}{x}\cdot\frac{1}{x+3}\to 1\cdot\frac{1}{3}$.

Chapter Summary

In this chapter we have reviewed differentiation. We've defined the derivative as the instantaneous rate of change of a function, and looked at estimating derivatives using tables and graphs. We've reviewed the formulas for derivatives of basic functions, as well as the product, quotient, and chain rules. We've looked at derivatives of implicitly defined functions and inverse functions, and reviewed two important theorems: Rolle's Theorem and the Mean Value Theorem.

For BC Calculus students, we've reviewed derivatives of parametrically defined functions and the use of L'Hopital's Rule for evaluating limits of indeterminate forms.

Practice Exercises

Part A. Directions: Answer these questions *without* using your calculator.

In each of Questions 1–20 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

#1. Product Rule
$5x^4 \tan x + x^5 \sec^2 x$

1. $y = x^5 \tan x$

(A) $5x^4 \tan x$ (B) $x^5 \sec^2 x$ (C) $5x^4 \sec^2 x$

(D) $5x^4 + \sec^2 x$ (E) $5x^4 \tan x + x^5 \sec^2 x$

#2 Quotient Rule
$\frac{(3x+1)(-1)-(2-x)(3)}{(3x+1)^2}$

$\frac{-3x-1-6+3x}{(3x+1)}$

2. $y = \frac{2-x}{3x+1}$ $\frac{\text{lodhi} - \text{hi dlo}}{\text{lo}^2}$

(A) $-\frac{7}{(3x+1)^2}$ (B) $\frac{6x-5}{(3x+1)^2}$ (C) $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ (E) $\frac{7-6x}{(3x+1)^2}$

#3 Chain Rule
$\frac{1}{2}(3-2x)^{-1/2} \cdot (-2)$

$-\frac{1}{\sqrt{3-2x}}$

3. $y = \sqrt{3-2x}$ $= (3-2x)^{1/2}$

(A) $\frac{1}{2\sqrt{3-2x}}$ (B) $-\frac{1}{\sqrt{3-2x}}$ (C) $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ (E) $\frac{2}{3}(3-2x)^{3/2}$

4. $y = \frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ (B) $-30(5x+1)^{-4}$ (C) $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ (E) $\frac{30}{(5x+1)^4}$

Power Rule

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

$\frac{dy}{dx} = \cancel{3}\left(\frac{2}{\cancel{3}}x^{-1/3}\right) - 4\left(\frac{1}{2}x^{-1/2}\right) - 0 = 2x^{-1/3} - 2x^{-1/2}$

(A) $2x^{1/3} - 2x^{-1/2}$ **(B)** $3x^{-1/3} - 2x^{-1/2}$ **(C)** $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ **(E)** $2x^{-1/3} - 2x^{-1/2}$

Convert to power 1st

6. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}} = 2x^{1/2} - \frac{1}{2}x^{-1/2}$

Power Rule

$\frac{dy}{dx} = \cancel{2}\left(\frac{1}{\cancel{2}}x^{-1/2}\right) - \frac{1}{2}\left(-\frac{1}{2}x^{-3/2}\right)$

$= x^{-1/2} + \frac{1}{4}x^{-3/2}$

$= \frac{1}{\sqrt{x}} + \frac{1}{4}$

(A) $x + \frac{1}{x\sqrt{x}}$ **(B)** $x^{-1/2} + x^{-3/2}$ **(C)** $\frac{4x-1}{4x\sqrt{x}}$

(D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ **(E)** $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

---

7. $y = \sqrt{x^2 + 2x - 1} = (x^2+2x+1)^{1/2}$

Chain Rule

$= \frac{1}{2}(x^2+2x+1)^{-1/2}(2x+2)$

$= \frac{2x+2}{2(x^2+2x+1)^{1/2}} = \frac{\cancel{2}(x+1)}{\cancel{2}\underbrace{\sqrt{x^2+2x+1}}_{y}} = \frac{x+1}{y}$

(A) $\frac{x+1}{y}$ **(B)** $4y(x+1)$ **(C)** $\frac{1}{2\sqrt{x^2+2x-1}}$

(D) $-\frac{x+1}{(x^2+2x-1)^{3/2}}$ **(E)** none of these

---

8. $y = \frac{x^2}{\cos x}$

Quotient Rule

$= \frac{\cos x\,(2x) - x^2(-\sin x)}{\cos^2 x}$

$= \frac{2x\cos x + x^2\sin x}{\cos^2 x}$

~~**(A)**~~ $\frac{2x}{\sin x}$ ~~**(B)**~~ $-\frac{2x}{\sin x}$ ~~**(C)**~~ $\frac{2x\cos x - x^2\sin x}{\cos^2 x}$

(D) $\frac{2x\cos x + x^2\sin x}{\cos^2 x}$ ~~**(E)**~~ $\frac{2x\cos x + x^2\sin x}{\sin^2 x}$

---

9. $y = \ln\left(\overset{u}{\frac{e^x}{e^x - 1}}\right)$

Chain Rule + Quotient Rule

$\frac{1}{u}\cdot\frac{dy}{dx} = \frac{1}{\frac{e^x}{e^x-1}} \cdot \frac{(e^x-1)(e^x) - e^x(e^x)}{(e^x-1)^2}$

Keep flip mult

$= \frac{e^x-1}{e^x} \cdot \frac{e^x[(e^x-1) - e^x]}{(e^x-1)^2}$

$= \frac{e^x - 1 - e^x}{e^x - 1} = \frac{-1}{e^x-1}$

(A) $x - \frac{e^x}{e^x - 1}$ **(B)** $\frac{1}{e^x - 1}$ **(C)** $-\frac{1}{e^x - 1}$

(D) 0 **(E)** $\frac{e^x - 2}{e^x - 1}$

---

10. $y = \tan^{-1}\frac{x}{2}$

(A) $\frac{4}{4+x^2}$ **(B)** $\frac{1}{2\sqrt{4-x^2}}$ **(C)** $\frac{2}{\sqrt{4-x^2}}$

(D) $\frac{1}{2+x^2}$ **(E)** $\frac{2}{x^2+4}$

Chain Rule

u

$\frac{1}{\sec x + \tan x} \cdot (\sec \tan x + \sec^2 x) = \frac{\sec x(\tan x + \sec x)}{\cancel{(\sec x + \tan x)}}$

11. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ (B) $\frac{1}{\sec x}$ (C) $\tan x + \frac{\sec^2 x}{\tan x}$

(D) $\frac{1}{\sec x + \tan x}$ (E) $-\frac{1}{\sec x + \tan x}$

12. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 (B) 1 (C) $\frac{2}{(e^x + e^{-x})^2}$

(D) $\frac{4}{(e^x + e^{-x})^2}$ (E) $\frac{1}{e^{2x} + e^{-2x}}$

13. $y = \ln\left(\sqrt{x^2+1}\right)$

(A) $\frac{1}{\sqrt{x^2+1}}$ (B) $\frac{2x}{\sqrt{x^2+1}}$ (C) $\frac{1}{2\left(x^2+1\right)}$

(D) $\frac{x}{x^2+1}$ (E) $\frac{2x}{x^2+1}$

14. $y = \sin\left(\frac{1}{x}\right)$

(A) $\cos\left(\frac{1}{x}\right)$ (B) $\cos\left(-\frac{1}{x^2}\right)$ (C) $-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

(D) $-\frac{1}{x^2}\sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right)$ (E) $\cos(\ln x)$

15. $y = \frac{1}{2 \sin 2x}$

(A) $-\csc 2x \cot 2x$ (B) $\frac{1}{4 \cos 2x}$ (C) $-4 \csc 2x \cot 2x$

(D) $\frac{\cos 2x}{2\sqrt{\sin 2x}}$ (E) $-\csc^2 2x$

Product Rule

16. $y = e^{-x} \cos 2x$

$\frac{dy}{dx} = e^{-x}\left[-\sin(2x) \cdot 2\right] + e^{-x}(-1)\cos 2x$

$= -e^{-x}\left[2 \sin 2x + \cos 2x\right]$

(A) $-e^{-x}(\cos 2x + 2 \sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x} \sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x} \sin 2x$

Chain Rule

17. $y = \sec^2(x) = [\sec x]^2$

$\frac{dy}{dx} = 2\sec x \cdot (\sec x \tan x)$

(A) $2 \sec x$ (B) $2 \sec x \tan x$ (C) $2 \sec^2 x \tan x$ $= 2\sec^2 x \tan x$

(D) $\sec^2 x \tan^2 x$ (E) $\tan x$

Product Rule + Chain Rule

18. $y = x \ln^3 x = x(\ln x)^3$

$\frac{dy}{dx} = x \cdot 3(\ln x)^2 \cdot \frac{1}{x} + \ln^3 x$

(A) $\frac{3 \ln^2 x}{x}$ (B) $3 \ln^2 x$ (C) $3x \ln^2 x + \ln^3 x$ $= 3\ln^2 x + \ln^3 x$

(D) $3(\ln x + 1)$ (E) none of these

Quotient Rule

19. $y = \frac{1+x^2}{1-x^2}$

$\frac{dy}{dx} = \frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2} = \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2}$

(A) $-\frac{4x}{(1-x^2)^2}$ (B) $\frac{4x}{(1-x^2)^2}$ (C) $\frac{-4x^3}{(1-x^2)^2}$ $= \frac{4x}{(1-x^2)^2}$

(D) $\frac{2x}{1-x^2}$ (E) $\frac{4}{1-x^2}$

20. $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\frac{1}{2\sqrt{1-x^2}}$ (B) $\frac{2}{\sqrt{1-x^2}}$ (C) $\frac{1+x}{\sqrt{1-x^2}}$

(D) $\frac{x^2}{\sqrt{1-x^2}}$ (E) $\frac{1}{\sqrt{1+x}}$

In each of Questions 21–24, y is a differentiable function of x. Choose the alternative that is the derivative $\frac{dy}{dx}$.

Implicit

21. $x^3 - y^3 = 1$

$3x^2 - 3y^2 \frac{dy}{dx} = 0$ solve for $\frac{dy}{dx}$

$3x^2 = 3y^2 \frac{dy}{dx}$

$\frac{\cancel{3}x^2}{\cancel{3}y^2} = \frac{dy}{dx}$

(A) x (B) $3x^2$ (C) $\sqrt[3]{3x^2}$ (D) $\frac{x^2}{y^2}$ (E) $\frac{3x^2-1}{y^2}$

Implicit + chain

22. $x + \cos(x + y) = 0$

$1 - \sin(x+y) \cdot (1 + \frac{dy}{dx}) = 0$

$\frac{1}{\sin(x+y)} = \frac{\sin(x+y)[1 + \frac{dy}{dx}]}{\sin(x+y)}$

$\frac{1}{\sin(x+y)} = 1 + \frac{dy}{dx}$

(A) $\csc(x + y) - 1$ (B) $\csc(x + y)$ (C) $\frac{x}{\sin(x+y)}$

(D) $\frac{1}{\sqrt{1-x^2}}$ (E) $\frac{1-\sin x}{\sin y}$

Implicit

23. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ (B) $-\cot y$ (C) $\frac{\cos x}{\sin y}$

(D) $-\csc y \cos x$ (E) $\frac{2-\cos x}{\sin y}$

$\cos x + \sin y \frac{dy}{dx} - 0 = 0$

$\frac{\sin y \frac{dy}{dx}}{\sin y} = \frac{-\cos x}{\sin y}$

$\frac{dy}{dx} = -\frac{\cos x}{\sin y}$

$6x - 2\left[x\frac{dy}{dx} + y\right] + 10y\frac{dy}{dx} = 0$ Product

24. $3x^2 - 2(xy) + 5y^2 = 1$

$6x - 2x\frac{dy}{dx} - 2y + 10y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left[10y - 2x\right] = 2y - 6x$

$\frac{dy}{dx} = \frac{\cancel{2}(y-3x)}{\cancel{2}(5y-x)}$

(A) $\dfrac{3x+y}{x-5y}$ (B) $\dfrac{y-3x}{5y-x}$ (C) $3x+5y$

(D) $\dfrac{3x+4y}{x}$ (E) none of these

BC ONLY

25. If $x = t^2 + 1$ and $y = 2t^3$, then $\dfrac{dy}{dx} =$

(A) $3t$ (B) $6t^2$ (C) $\dfrac{6t^2}{t^2+1}$ (D) $\dfrac{6t^2}{(t^2+1)^2}$ (E) $\dfrac{2t^4+6t^2}{(t^2+1)^2}$

26. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ (B) $\{0, -1, -2\}$ (C) $\{-1, +2\}$
(D) $\{0\}$ (E) $\{0, 1, 2\}$

#27
$f'(x) = 16\left(\frac{1}{2}x^{-1/2}\right) = 8x^{-1/2}$

$f''(x) = 8\left(-\frac{1}{2}x^{-3/2}\right) = -4x^{-3/2}$

$f''(4) = \frac{-4}{\sqrt{4^3}} = \frac{-4}{8} = -\frac{1}{2}$

27. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

$16x^{1/2}$

(A) -32 (B) -16 (C) -4 (D) -2 (E) $-\dfrac{1}{2}$

28. If $f(x) = \ln x^3$, then $f''(3)$ is

(A) $-\dfrac{1}{3}$ (B) -1 (C) -3 (D) 1 (E) none of these

#29
$2x + 2y\frac{dy}{dx} = 0$; $\frac{dy}{dx} = -\frac{x}{y}$

Quotient Rule

$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2} = \frac{-y + x\frac{dy}{dx}}{y^2} = \frac{-y - \frac{x^2}{y}}{y^2}\Big|_{(0,5)} = \frac{-5-0}{25}$

29. If a point moves on the curve $x^2 + y^2 = 25$, then, at $(0, 5)$, $\dfrac{d^2y}{dx^2}$ is

(A) 0 (B) $\dfrac{1}{5}$ (C) -5 (D) $-\dfrac{1}{5}$ (E) nonexistent

BC ONLY

30. If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then, when $t = 1$, $\dfrac{d^2y}{dx^2}$ is

(A) 1 (B) -1 (C) 0 (D) 3 (E) $\dfrac{1}{2}$

31. If $f(x) = 5^x$ and $5^{1.002} \simeq 5.016$, which is closest to $f'(1)$?

(A) 0.016 (B) 1.0 (C) 5.0 (D) 8.0 (E) 32.0

32. If $y = e^x(x - 1)$, then $y''(0)$ equals

(A) -2 (B) -1 (C) 0 (D) 1 (E) none of these

BC ONLY

33. If $x = e^\theta \cos\theta$ and $y = e^\theta \sin\theta$, then, when $\theta = \dfrac{\pi}{2}$, $\dfrac{dy}{dx}$ is

(A) 1 (B) 0 (C) $e^{\pi/2}$ (D) nonexistent (E) -1

BC ONLY

34. If $x = \cos t$ and $y = \cos 2t$, then $\dfrac{d^2y}{dx^2}$ $(\sin t \neq 0)$ is

(A) $4 \cos t$ **(B)** 4 **(C)** $\dfrac{4y}{x}$ **(D)** -4 **(E)** $-4 \cot t$

35. $\lim\limits_{h\to 0} \dfrac{(1+h)^6 - 1}{h}$ is

$f(x) = x^6$ $f'(1) = 6x^5 = 6(1)^5 = 6$

(A) 0 **(B)** 1 **(C)** 6 **(D)** ∞ **(E)** nonexistent

36. $\lim\limits_{h\to 0} \dfrac{\sqrt[3]{8+h} - 2}{h}$ is

$f(x) = x^{1/3}$ $f'(8) = \frac{1}{3}x^{-2/3} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{8^2}} = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

(A) 0 **(B)** $\dfrac{1}{12}$ **(C)** 1 **(D)** 192 **(E)** ∞

37. $\lim\limits_{h\to 0} \dfrac{\ln(e+h) - 1}{h}$ is

$f(x) = \ln x$ $f'(e) = \frac{1}{x} = \frac{1}{e}$

(A) 0 **(B)** $\dfrac{1}{e}$ **(C)** 1 **(D)** e **(E)** nonexistent

38. $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{x}$ is

$f(x) = \cos x$ $f'(0) = -\sin(0) = 0$

(A) -1 **(B)** 0 **(C)** 1 **(D)** ∞ **(E)** none of these

39. If $f(x) = \begin{cases} \dfrac{4x^2 - 4}{x - 1}, & x \neq 1 \\ 4, & x = 1 \end{cases}$, which of these statements are true?

$f(1) = 4$

$\lim\limits_{x\to 1} f(x) = \dfrac{4(x^2-1)}{x-1} = \dfrac{4(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 8$

I. $\lim\limits_{x\to 1} f(x)$ exists. ✓ $= 8$

II. f is continuous at $x = 1$. No b/c $\lim\limits_{x\to 1} f(x) = 8$ but $f(1) = 4$ (Not same) Removable Disc.

III. f is differentiable at $x = 1$. Not cont, thus not diff.

(A) none **(B)** I only **(C)** I and II only
(D) I and III only **(E)** I, II, and III

40. If $g(x) = \begin{cases} x^2, & x \leq 3 \\ 6x - 9, & x > 3 \end{cases}$, which of these statements are true?

① $f(3) = 9$
② $\lim\limits_{x\to 3^-} f(x) = 9$, $\lim\limits_{x\to 3^+} f(x) = 9$ — Sam
③ $\lim\limits_{x\to 3} = f(3)$ Continuous!

$g'(x) = \begin{cases} 2x \\ 6 \end{cases}$
① $f'(3) = 6$
② $\lim\limits_{x\to 3} = 6$

I. $\lim\limits_{x\to 3} g(x)$ exists. ✓

II. g is continuous at $x = 3$. ✓

III. g is differentiable at $x = 3$. ✓

(A) I only **(B)** II only **(C)** III only
(D) I and II only **(E)** I, II, and III

41. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

~~$f(-8) = \sqrt[3]{(-8)^2} = 4$~~
~~$f(8) = \sqrt[3]{8^2} = 4$~~

$f'(x) = \frac{2}{3}x^{-1/3} = \dfrac{2}{3x^{1/3}}$ $x \neq 0$

(A) $f(0)$ is not defined **(B)** $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist **(D)** $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist ✓

42. If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \leqslant x \leqslant \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line on that interval?

(A) 1 **(B)** -1 **(C)** $\sqrt{2}$ **(D)** 0 **(E)** nowhere

43. If h is the inverse function of f and if $f(x) = \dfrac{1}{x}$, then $h'(3) =$

(A) -9 **(B)** $-\dfrac{1}{9}$ **(C)** $\dfrac{1}{9}$ **(D)** 3 **(E)** 9

BC ONLY

Dominance

$\dfrac{\infty}{\#} = \infty$

44. $\lim\limits_{x\to\infty} \dfrac{e^x}{x^{50}}$ equals

(A) 0 **(B)** 1 **(C)** $\dfrac{1}{50!}$ **(D)** ∞ **(E)** none of these

45. If $\sin(xy) = x$, then $\dfrac{dy}{dx} =$

(A) $\sec(xy)$ **(B)** $\dfrac{\sec(xy)}{x}$ **(C)** $\dfrac{\sec(xy) - y}{x}$

(D) $\dfrac{1 + \sec(xy)}{x}$ **(E)** $\sec(xy) - 1$

46. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x}$ is

(A) 1 **(B)** 2 **(C)** $\dfrac{1}{2}$ **(D)** 0 **(E)** ∞

47. $\lim\limits_{x\to 0} \dfrac{\sin 3x}{\sin 4x}$ is

(A) 1 **(B)** $\dfrac{4}{3}$ **(C)** $\dfrac{3}{4}$ **(D)** 0 **(E)** nonexistent

48. $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{x}$ is

(A) nonexistent **(B)** 1 **(C)** 2 **(D)** ∞ **(E)** none of these

L'Hopital

$\dfrac{\sec^2(\pi x)\cdot\pi}{1} = \dfrac{\pi}{\cos^2(\pi x)}\Big|_{x=0} = \dfrac{\pi}{\cos^2(0)} = \pi$

49. $\lim\limits_{x\to 0} \dfrac{\tan \pi x}{x}$ is

(A) $\dfrac{1}{\pi}$ **(B)** 0 **(C)** 1 **(D)** π **(E)** ∞

50. $\lim\limits_{x\to\infty} x^2 \sin\dfrac{1}{x}$

(A) is 1 **(B)** is 0 **(C)** is ∞

(D) oscillates between -1 and 1 **(E)** is none of these the

BC ONLY

51. The graph in the xy-plane represented by $x = 3 + 2 \sin t$ and $y = 2 \cos t - 1$, for $-\pi \leqslant t \leqslant \pi$, is

(A) a semicircle **(B)** a circle **(C)** an ellipse
(D) half of an ellipse **(E)** a hyperbola

52. $\lim\limits_{x\to 0} \dfrac{\sec x - \cos x}{x^2}$ equals

(A) 0 **(B)** $\dfrac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** none of these

BC ONLY

In each of Questions 53–56 a pair of equations that represent a curve parametrically is given. Choose the alternative that is the derivative $\dfrac{dy}{dx}$.

53. $x = t - \sin t$ and $y = 1 - \cos t$

(A) $\dfrac{\sin t}{1-\cos t}$ **(B)** $\dfrac{1-\cos t}{\sin t}$ **(C)** $\dfrac{\sin t}{\cos t - 1}$

(D) $\dfrac{1-x}{y}$ **(E)** $\dfrac{1-\cos t}{t-\sin t}$

BC ONLY

54. $x = \cos^3 \theta$ and $y = \sin^3 \theta$

(A) $\tan^3 \theta$ **(B)** $-\cot \theta$ **(C)** $\cot \theta$ **(D)** $-\tan \theta$ **(E)** $-\tan^2 \theta$

55. $x = 1 - e^{-t}$ and $y = t + e^{-t}$

(A) $\dfrac{e^{-t}}{1-e^{-t}}$ **(B)** $e^{-t} - 1$ **(C)** $e^t + 1$ **(D)** $e^t - e^{-2t}$ **(E)** $e^t - 1$

56. $x = \dfrac{1}{1-t}$ and $y = 1 - \ln(1-t)$ $(t < 1)$

(A) $\dfrac{1}{1-t}$ **(B)** $t - 1$ **(C)** $\dfrac{1}{x}$ **(D)** $\dfrac{(1-t)^2}{t}$ **(E)** $1 + \ln x$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

ıese

In Questions 57–64, differentiable functions f and g have the values shown in the table.

x	f	f'	g	g'
0	2	1	5	–4
1	3	2	3	–3
2	5	3	1	–2
3	10	4	0	–1

57. If $A = f + 2g$, then $A'(3) =$ $f'(3) + 2g'(3) = 4 + 2(-1) = 2$

(A) -2 (B) 2 (C) 7 (D) 8 (E) 10

Product Rule

58. If $B = f \cdot g$, then $B'(2) =$ $f'(2)g(2) + f(2)g'(2) = 3(1) + 5(-2)$
-7

(A) -20 (B) -7 (C) -6 (D) -1 (E) 13

Quotient Rule

59. If $D = \frac{1}{g}$, then $D'(1) =$ $\frac{g(1)(0) - (1)g'(1)}{[g(1)]^2} = \frac{0-(-3)}{3^2} = \frac{3}{9} = \frac{1}{3}$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

Chain Rule

60. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$ $\frac{1}{2}[f(3)]^{-1/2}[f'(3)] = \frac{1}{2}\left[\frac{1}{\sqrt{10}}\right][4] = \frac{2}{\sqrt{10}}$

$= [f(x)]^{1/2}$

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{10}}$ (C) 2 (D) $\frac{2}{\sqrt{10}}$ (E) $4\sqrt{10}$

61. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

(A) $\frac{-13}{25}$ (B) $-\frac{1}{4}$ (C) $\frac{13}{25}$ (D) $\frac{13}{16}$ (E) $\frac{22}{25}$

Chain Rule

62. If $M(x) = f(g(x))$, then $M'(1) =$ $f'(g(1)) \cdot g'(1) = f'(3)g'(1) = 4 \cdot (-3)$
$= -12$

(A) -12 (B) -6 (C) 4 (D) 6 (E) 12

63. If $P(x) = f(x^3)$, then $P'(1) =$

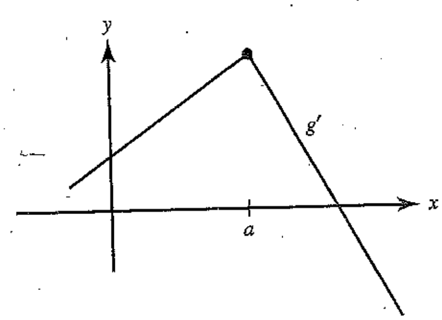
(A) 2 (B) 6 (C) 8 (D) 12 (E) 54

64. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) -2 (B) $-\frac{1}{25}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 2

65. The graph of g' is shown here. Which of the following statements is (are) true of g at $x = a$?

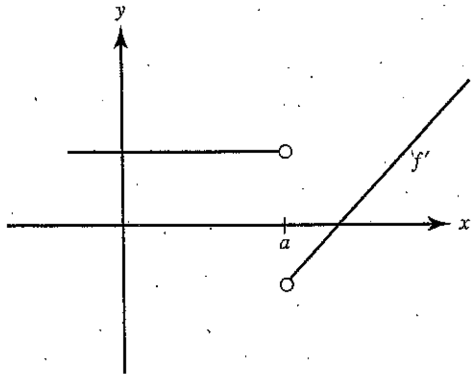
I. g is continuous. ✓
II. g is differentiable. ✓
III. g is increasing. ✓ b/c (+)



(A) I only (B) III only (C) I and III only
(D) II and III only (E) I, II, and III

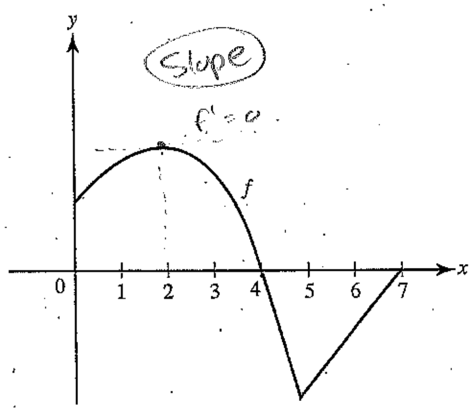
66. A function f has the derivative shown. Which of the following statements must be false?

(A) f is continuous at $x = a$.
(B) $f(a) = 0$.
(C) f has a vertical asymptote at $x = a$.
(D) f has a jump discontinuity at $x = a$.
(E) f has a removable discontinuity at $x = a$.



67. The function f whose graph is shown has $f' = 0$ at $x =$

(A) 2 only
(B) 2 and 5
(C) 4 and 7
(D) 2, 4, and 7
(E) 2, 4, 5, and 7



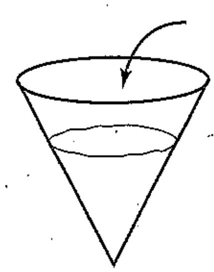
68. A differentiable function f has the values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

(A) 8 (B) 12 (C) 18 (D) 40 (E) 80

69. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is

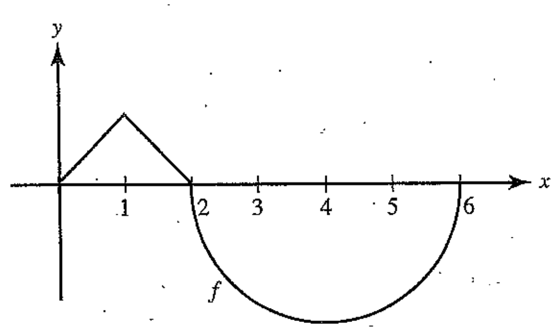
(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing



Use the figure to answer Questions 70–72. The graph of f consists of two line segments and a semicircle.

70. $f'(x) = 0$ for $x =$

(A) 1 only
(B) 2 only
(C) 4 only
(D) 1 and 4
(E) 2 and 6



71. $f'(x)$ does not exist for $x =$

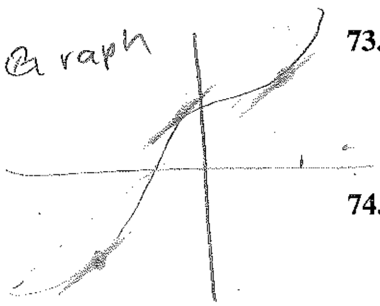
(A) 1 only (B) 2 only (C) 1 and 2
(D) 2 and 6 (E) 1, 2, and 6

72. $f'(5) =$

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) 1 (D) 2 (E) $\sqrt{3}$

#73

Graph



73. At how many points on the interval $[-5,5]$ is a tangent to $y = x + \cos x$ parallel to the secant line?

(A) none (B) 1 (C) 2 (D) 3 (E) more than 3

74. From the values of f shown, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	4.40	4.10	4.00

(A) -0.10 (B) -0.20 (C) -5 (D) -10 (E) -25

74

$\frac{f(2)-f(1.98)}{2-1.98} = \frac{4-4.10}{0.02}$

75. Using the values shown in the table for Question 74, estimate $(f^{-1})'(4)$.

(A) -0.2 (B) -0.1 (C) -5 (D) -10 (E) -25

76. The "left half" of the parabola defined by $y = x^2 - 8x + 10$ for $x \leq 4$ is a one-to-one function; therefore its inverse is also a function. Call that inverse g. Find $g'(3)$.

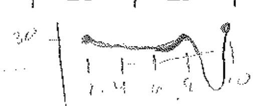
(A) $-\frac{1}{2}$ (B) $-\frac{1}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$ (E) $\frac{11}{2}$

77. The table below shows some points on a function f that is both continuous and differentiable on the closed interval $[2,10]$.

x	2	4	6	8	10
$f(x)$	30	25	20	25	30

Which must be true?

A.)



B.) Not necessarily @ $x=6$ see A.)

(A) $f(x) > 0$ for $2 < x < 10$ X
(B) $f'(6) = 0$ ✓
(C) $f'(8) > 0$
(D) The maximum value of f on the interval $[2,10]$ is 30.
(E) For some value of x on the interval $[2,10]$ $f'(x) = 0$.

C.) Not necessarily at 8 see A.)

78. If f is differentiable and difference quotients overestimate the slope of f at $x = a$ for all $h > 0$, which must be true?

(A) $f'(a) > 0$ (B) $f'(a) < 0$ (C) $f''(a) > 0$
(D) $f''(a) < 0$ (E) none of these

79. If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, then $(f \circ g)'(3)$ equals

(A) 0 (B) 1 (C) 6 (D) 9 (E) none of these

80. If $f(x) = \dfrac{x}{(x-1)^2}$, then the set of x's for which $f'(x)$ exists is

(A) all reals
(B) all reals except $x = 1$ and $x = -1$
(C) all reals except $x = -1$
(D) all reals except $x = \dfrac{1}{3}$ and $x = -1$
(E) all reals except $x = 1$

BC ONLY

81. If $y = \sqrt{x^2+1}$, then the derivative of y^2 with respect to x^2 is

(A) 1 (B) $\dfrac{x^2+1}{2x}$ (C) $\dfrac{x}{2(x^2+1)}$ (D) $\dfrac{2}{x}$ (E) $\dfrac{x^2}{x^2+1}$

BC ONLY

82. If $y = x^2 + x$, then the derivative of y with respect to $\dfrac{1}{1-x}$ is

(A) $(2x+1)(x-1)^2$ (B) $\dfrac{2x+1}{(1-x)^2}$ (C) $2x+1$

(D) $\dfrac{3-x}{(1-x)^3}$ (E) none of these

83. If $f(x) = \dfrac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $\dfrac{-\sqrt{x}}{(x^2+1)^2}$ (B) $-(x+1)^{-2}$ (C) $\dfrac{-2x}{(x^2+1)^2}$

(D) $\dfrac{1}{(x+1)^2}$ (E) $\dfrac{1}{2\sqrt{x}(x+1)}$

84. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

85. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

(A) -1 (B) $\dfrac{1}{5}$ (C) $\dfrac{1}{3}$ (D) 1 (E) 3

86. Suppose $f(1) = 2, f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

(A) equals $-\dfrac{1}{3}$ (B) equals $-\dfrac{1}{4}$ (C) equals $\dfrac{1}{4}$

(D) equals $\dfrac{1}{3}$ (E) cannot be determined

87. If $f(x) = x^3 - 3x^2 + 8x + 5$ and $g(x) = f^{-1}(x)$, then $g'(5) =$

(A) 8 (B) $\dfrac{1}{8}$ (C) 1 (D) $\dfrac{1}{53}$ (E) 53

99. How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

(A) none (B) 2 (C) 3 (D) 4 (E) 5

100. For $-6 < x < -3, f'(x)$ equals

(A) $-\frac{3}{2}$ (B) -1 (C) 1 (D) $\frac{3}{2}$ (E) 2

101. Which of the following statements about the graph of $f'(x)$ is false?

(A) It consists of six horizontal segments.
(B) It has four jump discontinuities.
(C) $f'(x)$ is discontinuous at each x in the set $\{-3,-1,1,2,5\}$.
(D) $f'(x)$ ranges from -3 to 2.
(E) On the interval $-1 < x < 1, f'(x) = -3$.

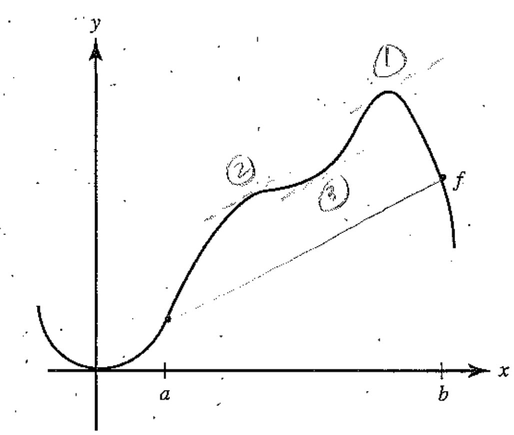
102. The table gives the values of a function f that is differentiable on the interval $[0,1]$:

x	0.10	0.20	0.30	0.40	0.50	0.60
$f(x)$	0.171	0.288	0.357	0.384	0.375	0.336

According to this table, the best approximation of $f'(0.10)$ is

(A) 0.12 (B) 1.08 (C) 1.17 (D) 1.77 (E) 2.88

103. At how many points on the interval $[a,b]$ does the function graphed satisfy the Mean Value Theorem?



(A) none (B) 1 (C) 2 (D) 3 (E) 4

Answer Key

1. E	**22.** A	**43.** B	**64.** D	**85.** C
2. A	**23.** D	**44.** D	**65.** E	**86.** D
3. B	**24.** B	**45.** C	**66.** C	**87.** B
4. B	**25.** A	**46.** B	**67.** A	**88.** D
5. E	**26.** E	**47.** C	**68.** D	**89.** B
6. D	**27.** E	**48.** E	**69.** E	**90.** D
7. A	**28.** A	**49.** D	**70.** C	**91.** C
8. D	**29.** D	**50.** C	**71.** E	**92.** B
9. C	**30.** E	**51.** B	**72.** B	**93.** E
10. E	**31.** D	**52.** C	**73.** D	**94.** B
11. A	**32.** D	**53.** A	**74.** C	**95.** D
12. D	**33.** E	**54.** D	**75.** A	**96.** E
13. D	**34.** B	**55.** E	**76.** B	**97.** A
14. C	**35.** C	**56.** C	**77.** E	**98.** E
15. A	**36.** B	**57.** B	**78.** C	**99.** E
16. A	**37.** B	**58.** B	**79.** C	**100.** D
17. C	**38.** B	**59.** E	**80.** E	**101.** B
18. E	**39.** B	**60.** D	**81.** A	**102.** C
19. B	**40.** E	**61.** C	**82.** A	**103.** D
20. C	**41.** E	**62.** A	**83.** B	
21. D	**42.** A	**63.** B	**84.** B	

Answers Explained

Many of the explanations provided include intermediate steps that would normally be reached on the way to a final algebraically simplified result. You may not need to reach the final answer.

NOTE: the formulas or rules cited in parentheses in the explanations are given on pages 115 and 116.

1. (E) By the Product Rule, (5),

$$y' = x^5(\tan x)' + (x^5)'\ (\tan x).$$

2. (A) By the Quotient Rule, (6),

$$y' = \frac{(3x+1)(-1)-(2-x)(3)}{(3x+1)^2} = -\frac{7}{(3x+1)^2}.$$

3. (B) Since $y = (3-2x)^{1/2}$, by the Power Rule, (3),

$$y' = \frac{1}{2}(3-3x)^{-1/2}\cdot(-2) = -\frac{1}{\sqrt{3-2x}}.$$

4. (B) Since $y = 2(5x+1)^{-3}, y' = -6(5x+1)^{-4}(5)$.

5. (E) $y' = 3\left(\frac{2}{3}\right)x^{-1/3} - 4\left(\frac{1}{2}\right)x^{-1/2}$

6. (D) Rewrite: $y = 2x^{1/2} - \frac{1}{2}x^{-1/2}$, so $y' = x^{-1/2} + \frac{1}{4}x^{-3/2}$.

Practice Exercises

#1) $3y^2 \frac{dy}{dx} - (y^2 + 2xy\frac{dy}{dx}) = 0$

$3y^2 \frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - 2xy) = y^2$

$\frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy}\Big|_{(1,2)} = \frac{4}{12-4} = \frac{1}{2}$

Part A. Directions: Answer these questions *without* using your calculator.

Product Rule

$8 - 4x = 4$, $4 = 4x$, $x = 1$

1. The slope of the curve $y^3 - xy^2 = 4$ at the point where $y = 2$ is

(A) -2 (B) $\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 2

2. The slope of the curve $y^2 - xy - 3x = 1$ at the point $(0, -1)$ is

(A) -1 (B) -2 (C) $+1$ (D) 2 (E) -3

$\boxed{\text{Product Rule}}$

#3) $\frac{dy}{dx} = x\cos x + \sin x \Big|_{x=\pi/2}$

$= \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}$

$= 1$ (slope)

$y - \frac{\pi}{2} = 1(x - \frac{\pi}{2})$

$y = x - \frac{\pi}{2} + \frac{\pi}{2}$

3. The equation of the tangent to the curve $y = x\sin x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(A) $y = x - \pi$ (B) $y = \frac{\pi}{2}$ (C) $y = \pi - x$

(D) $y = x + \frac{\pi}{2}$ (E) $y = x$

$y' = 0$

4. The tangent to the curve of $y = xe^{-x}$ is horizontal when x is equal to

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{e}$ (E) none of these

$\boxed{\text{Product Rule}}$

#4) $y' = x(-e^{-x}) + e^{-x}$

$= e^{-x}(-x + 1) = 0$ when $\boxed{x = 1}$

5. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is

(A) 0 (B) 2 (C) 6 (D) -2 (E) none of these

#5) $\frac{dy}{dx} = 0$ critical pts possible min/max

$y' \to 5x^4 + 3x^2 - 2 = 0$ $a \cdot c = -10$, $b = 3$

$5x^4 - 2x^2 + 5x^2 - 2 = 0$

$x^2(5x^2 - 2) + 1(5x^2 - 2) = 0$

$(x^2 + 1)(5x^2 - 2) = 0$

$x = \sqrt{2/5}$ min value

6. The equation of the tangent to the hyperbola $x^2 - y^2 = 12$ at the point $(4, 2)$ on the curve is

(5, 2)

(A) $x - 2y + 6 = 0$ (B) $y = 2x$ (C) $y = 2x - 6$

(D) $y = \frac{x}{2}$ (E) $x + 2y = 6$

$y'' = 20x^3 + 6x\big|_{x=\sqrt{2/5}}$

$= -2$

Now need slope at this pt

$\sqrt{2/5}$

7. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(A) $y = 0$ (B) $y = \pm\sqrt{3}$ (C) $y = \frac{1}{2}$

(D) $y = \pm 3$ (E) none of these

#7) $2y\frac{dy}{dx} - (x\frac{dy}{dx} + y) + 0 = 0$

$2y\frac{dy}{dx} - x\frac{dy}{dx} - y = 0$

$\frac{dy}{dx}(2y - x) = y$

8. The best approximation, in cubic inches, to the increase in volume of a sphere when the radius is increased from 3 to 3.1 in. is

(A) $\frac{0.04\pi}{3}$ (B) 0.04π (C) 1.2π (D) 3.6π (E) 36π

$\frac{dy}{dx} = \frac{y}{2y - x}$ ← horizontal if $= 0$ ← vertical if denom $= 0$

9. When $x = 3$, the equation $2x^2 - y^3 = 10$ has the solution $y = 2$. When $x = 3.04$, $y \simeq$

(A) 1.6 (B) 1.96 (C) 2.04 (D) 2.14 (E) 2.4

$2y - x = 0$

$x = 2y$ plug in $y^2 - (2y)y + 9 = 0$

$-y^2 = -9$

$y^2 = 9$

$y = \pm 3$

10. If the side e of a square is increased by 1%, then the area is increased approximately

(A) $0.02e$ (B) $0.02e^2$ (C) $0.01e^2$ (D) 1% (E) $0.01e$

11. The edge of a cube has length 10 in., with a possible error of 1%. The possible error, in cubic inches, in the volume of the cube is

(A) 3 (B) 1 (C) 10 (D) 30 (E) none of these

12. The function $f(x) = x^4 - 4x^2$ has

Crit pts $f'(x) = 0$

$f'(x) = 4x^3 - 8x = 4x(x^2 - 4)$
$= 4x(x+2)(x-2) = 0$

$x = 0$ $x = \pm 2$

min max min

(A) one relative minimum and two relative maxima
(B) one relative minimum and one relative maximum
(C) two relative maxima and no relative minimum
(D) two relative minima and no relative maximum
(E) two relative minima and one relative maximum

13. The number of inflection points of the curve in Question 12 is

POI $f''(x) = 0$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$f''(x) = 12x^2 - 8 = 0$
$x = \pm\sqrt{8/12} = \pm\sqrt{2/3}$

CCU CCD CCU

$-\sqrt{2/3}$ $\sqrt{2/3}$

14. The maximum value of the function $y = -4\sqrt{2-x}$ is $= -4(2-x)^{1/2}$

(A) 0 (B) -4 (C) 2 (D) -2 (E) none of these

$\frac{dy}{dx} = 0$ crit pt $-4 \cdot \frac{1}{2}(2-x)^{-1/2}(-1) = 0$

$2(2-x)^{-1/2} = 0$

$\frac{2}{\sqrt{2-x}} = 0$ Never

Domain $x \geq 2$ but Endpt @ $x = 2$ is a max of (0)

15. The total number of local maximum and minimum points of the function whose derivative, for all x, is given by $f'(x) = x(x-3)^2(x+1)^4$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) none of these

#15

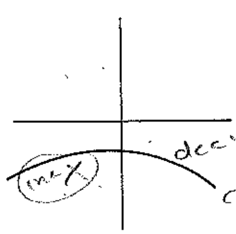
$x = 0$, $x = 3$, $x = -1$

1 min
0 max

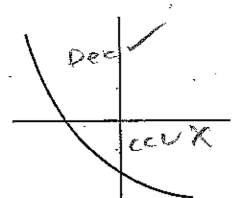
16. For which curve shown below are both f' and f'' negative?

Dec CCD

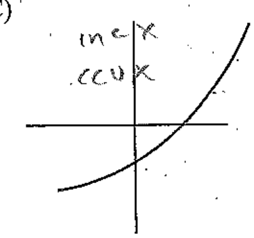
(A)



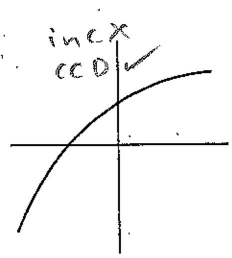
(B)



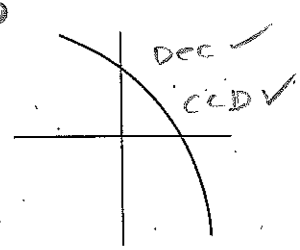
(C)



(D)



(E)



17. For which curve shown in question 16 is f'' positive but f' negative? (B)

CCU dec

In Questions 18–21, the position of a particle moving along a straight line is given by $s = t^3 - 6t^2 + 12t - 8$.

18. The distance s is increasing for

(A) $t < 2$ (B) all t except $t = 2$ (C) $1 < t < 3$
(D) $t < 1$ or $t > 3$ (E) $t > 2$

#19

$v(t) = s'$
$= 3t^2 - 12t + 12$
$3(t^2 - 4t + 4)$
$3(t-2)^2$
$v \geq 0$

19. The minimum value of the speed is

(A) 1 (B) 2 (C) 3 (D) 0 (E) none of these

20. The acceleration is positive

(A) when $t > 2$ (B) for all t, $t \neq 2$ (C) when $t < 2$
(D) for $1 < t < 3$ (E) for $1 < t < 2$

21. The speed of the particle is decreasing for

(A) $t > 2$ (B) $t < 3$ (C) all t
(D) $t < 1$ or $t > 2$ (E) none of these

In Questions 22–24, a particle moves along a horizontal line and its position at time t is $s = t^4 - 6t^3 + 12t^2 + 3$.

$v(t) = 4t^3 - 18t^2 + 24t$
$= 2t(2t^2 - 9t + 6)$
only if $t = 0$

22. The particle is at rest when t is equal to

(A) 1 or 2 (B) 0 (C) $\frac{9}{4}$ (D) 0, 2, or 3 (E) none of these

plug in answers

accel

$a(t) = 12t^2 - 36t + 24$
$= 12(t^2 - 3t + 2)$
$= 12(t-1)(t-2)$

23. The velocity, v, is increasing when

(A) $t > 1$ (B) $1 < t < 2$ (C) $t < 2$
(D) $t < 1$ or $t > 2$ (E) $t > 0$

24. The speed of the particle is increasing for

(A) $0 < t < 1$ or $t > 2$ (B) $1 < t < 2$ (C) $t < 2$
(D) $t < 0$ or $t > 2$ (E) $t < 0$

25. The displacement from the origin of a particle moving on a line is given by $s = t^4 - 4t^3$. The maximum displacement during the time interval $-2 \leqq t \leqq 4$ is

(A) 27 (B) 3 (C) $12\sqrt{3} + 3$
(D) 48 (E) none of these

$\frac{ds}{dt} = v(t) = 0$
$5t^4 + 20t^3 = 0$
$5t^3(t + 4) = 0$
$t = 0$ or $t = -4$

26. If a particle moves along a line according to the law $s = t^5 + 5t^4$, then the number of times it reverses direction is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

BC ONLY

In Questions 27–30, $\mathbf{R} = \left\langle 3 \cos \frac{\pi}{3}t,\ 2 \sin \frac{\pi}{3}t \right\rangle$ is the (position) vector $\langle x, y \rangle$ from the origin to a moving point $P(x, y)$ at time t.

27. A single equation in x and y for the path of the point is

(A) $x^2 + y^2 = 13$ (B) $9x^2 + 4y^2 = 36$ (C) $2x^2 + 3y^2 = 13$
(D) $4x^2 + 9y^2 = 1$ (E) $4x^2 + 9y^2 = 36$

28. When $t = 3$, the speed of the particle is

(A) $\frac{2\pi}{3}$ (B) 2 (C) 3 (D) π (E) $\frac{\sqrt{13}}{3}\pi$

29. Tne magnitude of the acceleration when $t = 3$ is

(A) 2 (B) $\frac{\pi^2}{3}$ (C) 3 (D) $\frac{2\pi^2}{9}$ (E) π

30. At the point where $t = \frac{1}{2}$, the slope of the curve along which the particle moves is

(A) $-\frac{2\sqrt{3}}{9}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{2}{\sqrt{3}}$

(D) $-\frac{2\sqrt{3}}{3}$ (E) none of these

31. A balloon is being filled with helium at the rate of 4 ft³/min. The rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ ft³ is

(A) 4π (B) 2 (C) 4 (D) 1 (E) 2π

$V = \frac{4}{3}\pi r^3$
$\frac{dV}{dt} = \frac{4}{3}(3\pi r^2)\frac{dr}{dt}$
$4 = 4\pi r^2 \frac{dr}{dt}$
$\frac{dr}{dt} = \frac{1}{\pi r^2}$
(Initial Condition)
$V = \frac{4}{3}\pi r^3$
$\frac{32\pi}{3} = \frac{4\pi}{3} r^3$
$8 = r^3$
$r = 2$

$S = 4\pi r^2$
$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$
$\frac{dS}{dt} = \frac{8\pi r}{1} \cdot \frac{1}{\pi r^2}$
$= \frac{8}{r}$
$= 4\ ft^2$

32. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of $\frac{1}{2}$ ft/hr. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

(A) 4π (B) 8π (C) 16π (D) $\frac{1}{4\pi}$ (E) $\frac{1}{8\pi}$

33. A local minimum value of the function $y = \frac{e^x}{x}$ is

(A) $\frac{1}{e}$ (B) 1 (C) -1 (D) e (E) 0

$y' = \frac{xe^x - e^x \cdot 1}{x^2}$
critical pts. $y' = 0$
$e^x(x-1) = 0$
$x = 1$
minimum value is
$y = \frac{e^1}{1} = e$

34. The area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve of $y = e^{-x^2}$ is

CHALLENGE

(A) $\sqrt{\frac{2}{e}}$ (B) $\sqrt{2e}$ (C) $\frac{2}{e}$ (D) $\frac{1}{\sqrt{2e}}$ (E) $\frac{2}{e^2}$

CHALLENGE

35. A line is drawn through the point (1, 2) forming a right triangle with the positive x- and y-axes. The slope of the line forming the triangle of least area is

(A) -1 (B) -2 (C) -4 (D) $-\frac{1}{2}$ (E) -3

36. The point(s) on the curve $x^2 - y^2 = 4$ closest to the point (6, 0) is (are)

(A) (2, 0) (B) $(\sqrt{5}, \pm 1)$ (C) $(3, \pm\sqrt{5})$

(D) $(\sqrt{13}, \pm\sqrt{3})$ (E) none of these

37. The sum of the squares of two positive numbers is 200; their minimum product is

No min

(A) 100 (B) $25\sqrt{7}$ (C) 28

(D) $24\sqrt{14}$ (E) none of these

#37 $x^2 + y^2 = 200$

$x\,y$ minimized

$y = \sqrt{200 - x^2}$

$x \cdot y = x\sqrt{200 - x^2} = x(200-x^2)^{1/2}$

derivative $= (200-x^2)^{1/2} + x \cdot \frac{1}{2}(200-x^2)^{-1/2} \cdot -2x$ (chain rule)

$= (200-x^2)^{1/2} - \frac{x^2}{(200-x^2)^{1/2}}$

$= \frac{200 - x^2 - x^2}{(200-x^2)^{1/2}}$

$= \frac{2(100-x^2)}{\sqrt{200-x^2}} = 0$ @ $x = \pm 10$

$+$ $+$ $-$; -11, -10, 0, 10, 11

Product Rule

38. The first-quadrant point on the curve $y^2x = 18$ that is closest to the point (2, 0) is

(A) (2, 3) (B) $(6, \sqrt{3})$ (C) $(3, \sqrt{6})$

(D) $(1, 3\sqrt{2})$ (E) none of these

39. If h is a small negative number, then the local linear approximation for $\sqrt[3]{27+h}$ is

(A) $3 + \frac{h}{27}$ (B) $3 - \frac{h}{27}$ (C) $\frac{h}{27}$

(D) $-\frac{h}{27}$ (E) $3 - \frac{h}{9}$

40. If $f(x) = xe^{-x}$, then at $x = 0$

(A) f is increasing (B) f is decreasing (C) f has a relative maximum

(D) f has a relative minimum (E) f' does not exist

#40 $f'(x) = xe^{-x}(-1) + e^{-x}$

$= e^{-x}(-x+1)$

$= \frac{1-x}{e^x}$

$f'(0) = \frac{1-0}{e^0} = 1$ (pos) inc

41. A function f has a derivative for each x such that $|x| < 2$ and has a local minimum at (2, –5). Which statement below must be true?

(A) $f'(2) = 0$.

(B) f' exists at $x = 2$.

(C) The graph is concave up at $x = 2$.

(D) $f'(x) < 0$ if $x < 2$, $f'(x) > 0$ if $x > 2$.

(E) None of the preceding is necessarily true.

42. The height of a rectangular box is 10 in. Its length increases at the rate of 2 in./sec; its width decreases at the rate of 4 in./sec. When the length is 8 in. and the width is 6 in., the rate, in cubic inches per second, at which the volume of the box is changing is

(A) 200 (B) 80 (C) -80 (D) -200 (E) -20

$l = 8$, $w = 6$, h, $\frac{dw}{dt} = -4$, $h = 10$

$\frac{dl}{dt} = 2$ in/s

$V = l \cdot w \cdot h$

$= 10\,[l\,w]$ product rule

$\frac{dV}{dt} = 10\left[l\frac{dw}{dt} + w\frac{dl}{dt}\right]$

$= 10[8(-4) + 6(2)]$

$= 10[-32 + 12]$

$= -200$

43. The tangent to the curve $x^3 + x^2y + 4y = 1$ at the point (3, –2) has slope

Product Rule

(A) -3 (B) $-\frac{23}{9}$ (C) $-\frac{27}{13}$ (D) $-\frac{11}{9}$ (E) $-\frac{15}{13}$

Implicit

$3x^2 + \left[2xy + x^2\frac{dy}{dx}\right] + 4\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left[x^2 + 4\right] = -3x^2 - 2xy$

$\frac{dy}{dx} = \frac{-3x^2 - 2xy}{x^2 + 4}\Big|_{(3,-2)}$

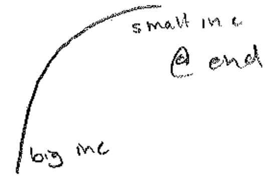
$= \frac{-27 + 12}{13} = \frac{-15}{13}$

44. If $f(x) = ax^4 + bx^2$ and $ab > 0$, then

(A) the curve has no horizontal tangents
(B) the curve is concave up for all x
(C) the curve is concave down for all x
(D) the curve has no inflection point
(E) none of the preceding is necessarily true

45. A function f is continuous and differentiable on the interval [0,4], where f' is positive but f'' is negative. Which table could represent points on f?

inc
CCD



(A)

x	0	1	2	3	4
y	10	12	14	16	18

X b/c inc const rate

(B)

x	0	1	2	3	4
y	10	12	15	19	24

X b/c inc big @ end

(C)

x	0	1	2	3	4
y	10	18	24	28	30

(D)

x	0	1	2	3	4
y	30	28	24	18	10

X b/c dec

(E)

x	0	1	2	3	4
y	24	19	15	12	10

X b/c dec

46. The equation of the tangent to the curve with parametric equations $x = 2t + 1, y = 3 - t^3$ at the point where $t = 1$ is

BC ONLY

(A) $2x + 3y = 12$ (B) $3x + 2y = 13$ (C) $6x + y = 20$
(D) $3x - 2y = 5$ (E) none of these

47. Approximately how much less than 4 is $\sqrt[3]{63}$?

(A) $\frac{1}{48}$ (B) $\frac{1}{16}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$ (E) 1

48. The best linear approximation for $f(x) = \tan x$ near $x = \frac{\pi}{4}$ is

(A) $1 + \frac{1}{2}(x - \frac{\pi}{4})$ (B) $1 + (x - \frac{\pi}{4})$ (C) $1 + \sqrt{2}\,(x - \frac{\pi}{4})$

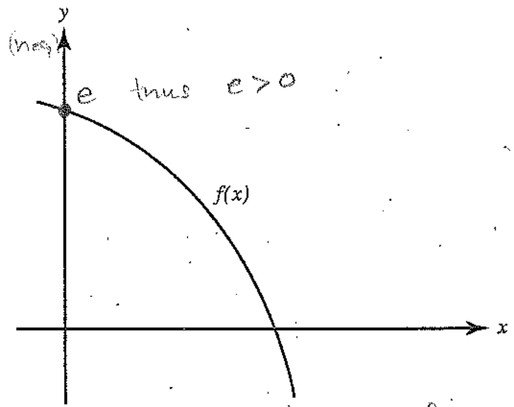
(D) $1 + 2(x - \frac{\pi}{4})$ (E) $2 + 2(x - \frac{\pi}{4})$

49. When h is near zero, e^{kh}, using the tangent-line approximation, is approximately

(A) k (B) kh (C) 1 (D) $1 + k$ (E) $1 + kh$

50. If $f(x) = cx^2 + dx + e$ for the function shown in the graph, then

#50
$f'(x) = 2cx + d$
$f''(x) = 2c$ since CCD $f''<0$ (neg)
thus $c<0$ or neg
$f'(0) = $ Neg
$f'(0) = d$ so $d<0$



(A) c, d, and e are all positive
(B) $c > 0, d < 0, e < 0$
(C) $c > 0, d < 0, e > 0$
(D) $c < 0, d > 0, e > 0$
(E) $c < 0, d < 0, e > 0$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

51. The point on the curve $y = \sqrt{2x+1}$ at which the normal is parallel to the line $y = -3x + 6$ is

(A) (4, 3) (B) (0, 1) (C) $(1, \sqrt{3})$
(D) (4, –3) (E) $(2, \sqrt{5})$

#51 $y = (2x+1)^{1/2}$
$\frac{dy}{dx} = \frac{1}{2}(2x+1)^{-1/2}(2)$
$= \frac{1}{\sqrt{2x+1}}$
parallel to $y=-3x+6$ $m=-3$ $m_\perp = \frac{1}{3}$
$\left(\frac{1}{\sqrt{2x+1}}\right)^2 = \left(\frac{1}{3}\right)^2$
$\frac{1}{2x+1} = \frac{1}{9}$
$2x+1 = 9$
$x = 4$
$y = \sqrt{2(4)+1} = 3$

52. The equation of the tangent to the curve $x^2 = 4y$ at the point on the curve where $x = -2$ is

(A) $x + y - 3 = 0$ (B) $y - 1 = 2x(x + 2)$ (C) $x - y + 3 = 0$
(D) $x + y - 1 = 0$ (E) $x + y + 1 = 0$

#52 $2x = 4\frac{dy}{dx}$
$\frac{dy}{dx} = \frac{x}{2}\Big|_{x=-2}$
slope $= -1$
$(-2)^2 = 4y$
$\frac{4}{4} = y$
$1 = y \rightarrow (-2, 1)$ point
$y - 1 = -1(x+2)$

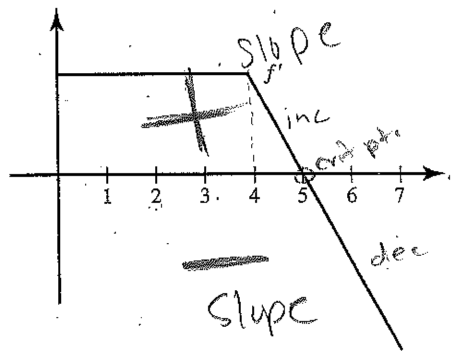
53. The table shows the velocity at time t of an object moving along a line. Estimate the acceleration (in ft/sec²) at $t = 6$ sec.

t (sec)	0	4	8	10
vel.	18	16	10	0

(A) –6 (B) –1.8 (C) –1.5 (D) 1.5 (E) 6

$a(t) = \frac{10-16}{8-4} = \frac{-6}{4} = -\frac{3}{2}$

Use the graph shown, sketched on [0, 7], for Questions 54–56.



54. From the graph it follows that

(A) f is discontinuous at $x = 4$ ✗ continuous
(B) f is decreasing for $4 < x < 7$ ✗ inc from 4 to 5, dec from 5 to 7
(C) f is constant for $0 < x < 4$ ✗ inc from 0 to 4
(D) f has a local maximum at $x = 0$ ✗
(E) f has a local minimum at $x = 7$ ✓

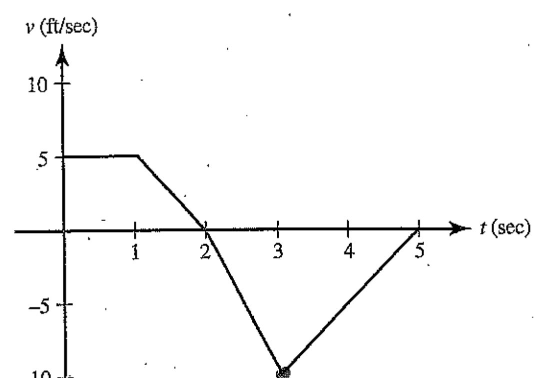
55. Which statement best describes f at $x = 5$? f' goes from pos to neg = local max

(A) f has a root. **(B)** f has a maximum. **(C)** f has a minimum.
(D) The graph of f has a point of inflection. **(E)** none of these

56. For which interval is the graph of f concave downward?

(A) (0,4) **(B)** (4,5) **(C)** (5,7)
(D) (4,7) **(E)** none of these

Use the graph shown for Questions 57–63. It shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 5$.



largest |v|

57. The object attains its maximum speed when $t =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5 moving away from x axis

58. The speed of the object is increasing during the time interval

(A) (0,1) **(B)** (1,2) **(C)** (0,2) **(D)** (2,3) **(E)** (3,5)

Slope

59. The acceleration of the object is positive during the time interval

(A) (0,1) **(B)** (1,2) **(C)** (0,2) **(D)** (2,3) **(E)** (3,5)

60. How many times on $0 < t < 5$ is the object's acceleration undefined?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

#61 $a(t) = \frac{-10-0}{3-2} = \frac{-10}{1}$

61. During $2 < t < 3$ the object's acceleration (in ft/sec²) is

(A) −10 **(B)** −5 **(C)** 0 **(D)** 5 **(E)** 10

#62 largest (+) occurs when t = 2

Area (+)

62. The object is furthest to the right when $t =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

$a(t)_{avg} = \frac{-10-5}{3-0} = \frac{-15}{3} = -5$

63. The object's average acceleration (in ft/sec²) for the interval $0 \leq t \leq 3$ is

(A) −15 **(B)** −5 **(C)** −3 **(D)** −1 **(E)** none of these

64. The line $y = 3x + k$ is tangent to the curve $y = x^3$ when k is equal to

(A) 1 or −1 **(B)** 0 **(C)** 3 or −3 **(D)** 4 or −4 **(E)** 2 or −2

65. The two tangents that can be drawn from the point (3,5) to the parabola $y = x^2$ have slopes

(A) 1 and 5 **(B)** 0 and 4 **(C)** 2 and 10

(D) 2 and $-\frac{1}{2}$ **(E)** 2 and 4

66. The table shows the velocity at various times of an object moving along a line. An estimate of its acceleration (in ft/sec²) at $t = 1$ is

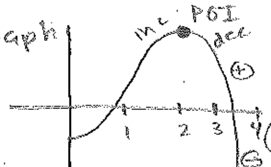
t (sec)	1.0	1.5	2.0	2.5
v (ft/sec)	12.2	13.0	13.4	13.7

(A) 0.8 **(B)** 1.0 **(C)** 1.2 **(D)** 1.4 **(E)** 1.6

#67 $f'(x) = x \sin x - \cos x$

Crit pts $f'(x) = 0$ or DNE

Graph



For Questions 67 and 68, $f'(x) = x \sin x - \cos x$ for $0 < x < 4$.

67. f has a local maximum when x is approximately

(A) 0.9 **(B)** 1.2 **(C)** 2.3 **(D)** 3.4 **(E)** 3.7

68. The graph of f has a point of inflection when x is approximately

(A) 0.9 **(B)** 1.2 **(C)** 2.3 **(D)** 3.4 **(E)** 3.7

Max of f' b/c f' goes from inc to dec

BC ONLY

In Questions 69–72, the motion of a particle in a plane is given by the pair of equations $x = 2t$ and $y = 4t - t^2$.

69. The particle moves along

(A) an ellipse **(B)** a circle **(C)** a hyperbola
(D) a line **(E)** a parabola

70. The speed of the particle at any time t is

(A) $\sqrt{6-2t}$ **(B)** $2\sqrt{t^2-4t+5}$ **(C)** $2\sqrt{t^2-2t+5}$

(D) $\sqrt{8}(|t-2|)$ **(E)** $2(|3-t|)$

71. The minimum speed of the particle is

(A) 2 **(B)** $2\sqrt{2}$ **(C)** 0 **(D)** 1 **(E)** 4

72. The acceleration of the particle

(A) depends on t
(B) is always directed upward
(C) is constant both in magnitude and in direction
(D) never exceeds 1 in magnitude
(E) is none of these

73. If a particle moves along a curve with constant speed, then

(A) the magnitude of its acceleration must equal zero
(B) the direction of acceleration must be constant
(C) the curve along which the particle moves must be a straight line
(D) its velocity and acceleration vectors must be perpendicular
(E) the curve along which the particle moves must be a circle

74. A particle is moving on the curve of $y = 2x - \ln x$ so that $\frac{dx}{dt} = -2$ at all times t. At the point (1,2), $\frac{dy}{dt}$ is

(A) 4 **(B)** 2 **(C)** −4 **(D)** 1 **(E)** −2

In questions 75–76, a particle is in motion along the polar curve $r = 6\cos 2\theta$ such that $\frac{d\theta}{dt} = \frac{1}{3}$ radian/sec when $\theta = \frac{\pi}{6}$.

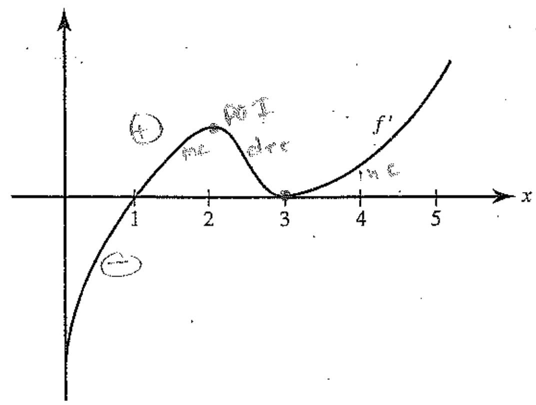
75. At that point, find the rate of change (in units per second) of the particle's distance from the origin.

(A) $-6\sqrt{3}$ **(B)** $-2\sqrt{3}$ **(C)** $-\sqrt{3}$ **(D)** $2\sqrt{3}$ **(E)** $6\sqrt{3}$

76. At that point, what is the horizontal component of the particle's velocity?

(A) $-\frac{21}{2}$ **(B)** $-\frac{7}{2}$ **(C)** -2 **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{2}$

Use the graph of f' on [0,5], shown below, for Questions 77 and 78.



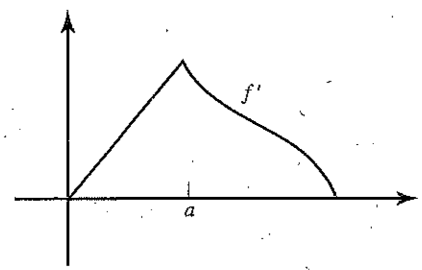
77. f has a local minimum at $x =$ b/c f' changes from − to +

(A) 0   (B) 1   (C) 2   (D) 3   (E) 5   min

78. The graph of f has a point of inflection at $x =$ b/c f' goes from inc to dec

(A) 1 only   (B) 2 only   (C) 3 only
(D) 2 and 3 only   (E) none of these

79. It follows from the graph of f', shown at the right, that



(A) f is not continuous at $x = a$
(B) f is continuous but not differentiable at $x = a$
(C) f has a relative maximum at $x = a$
(D) The graph of f has a point of inflection at $x = a$
(E) none of these

80. A vertical circular cylinder has radius r ft and height h ft. If the height and radius both increase at the constant rate of 2 ft/sec, then the rate, in square feet per second, at which the lateral surface area increases is

(A) $4\pi r$ (B) $2\pi(r + h)$ (C) $4\pi(r + h)$ (D) $4\pi rh$ (E) $4\pi h$

81. A tangent drawn to the parabola $y = 4 - x^2$ at the point (1, 3) forms a right triangle with the coordinate axes. The area of the triangle is

(A) $\frac{5}{4}$ (B) $\frac{5}{2}$ (C) $\frac{25}{2}$ (D) 1   (E) $\frac{25}{4}$

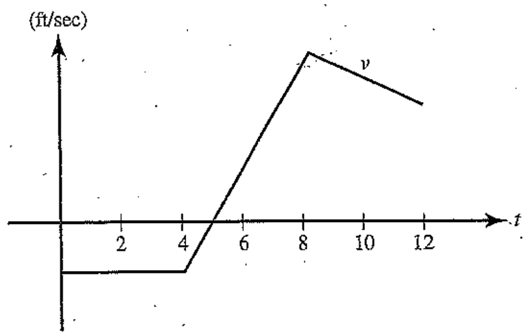
82. Two cars are traveling along perpendicular roads, car A at 40 mph, car B at 60 mph. At noon, when car A reaches the intersection, car B is 90 mi away, and moving toward it. At 1 P.M. the rate, in miles per hour, at which the distance between the cars is changing is

(A) −40   (B) 68   (C) 4   (D) −4   (E) 40

83. For Question 82, if t is the number of hours of travel after noon, then the cars are closest together when t is

(A) 0 (B) $\frac{27}{26}$ (C) $\frac{9}{5}$ (D) $\frac{3}{2}$ (E) $\frac{14}{13}$

The graph for Questions 84 and 85 shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 12$.



Since graph v, a(t) is slope
So greatest slope

84. For what t does this object attain its maximum acceleration?

(A) $0 < t < 4$ (B) $4 < t < 8$ (C) $t = 5$ (D) $t = 8$ (E) $t = 12$

85. The object reverses direction at $t =$

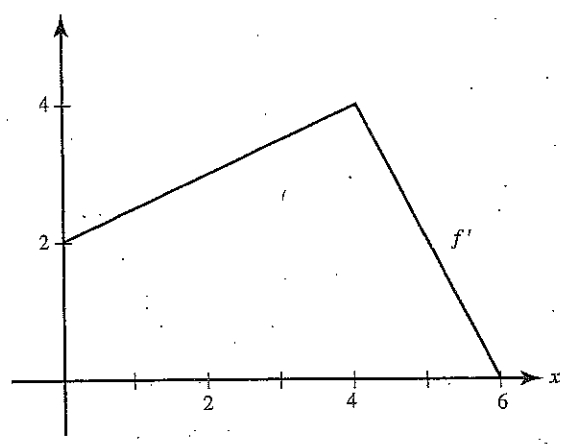
reverses direction occurs when move right (+v) to left (−v) or left (−v) to right (+v)

(A) 4 only (B) 5 only (C) 8 only
(D) 5 and 8 (E) none of these

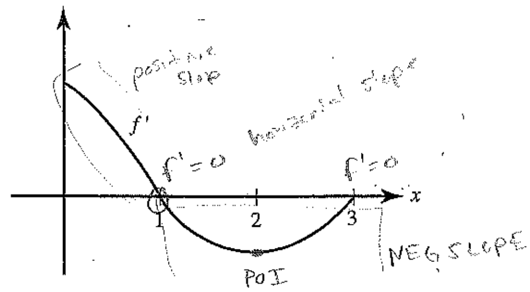
☆ Velocity changes sign ☆

86. The graph of f' is shown below. If we know that $f(2) = 10$, then the local linearization of f at $x = 2$ is $f(x) \simeq$

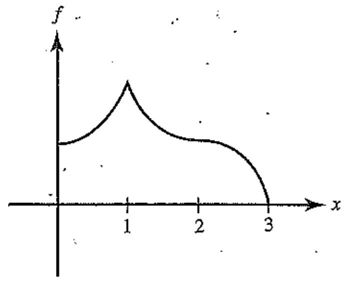
(A) $\frac{x}{2} + 2$ (B) $\frac{x}{2} + 9$ (C) $3x - 3$
(D) $3x + 4$ (E) $10x - 17$



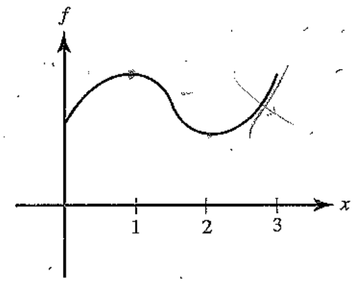
87. Given f' as graphed, which could be the graph of f?

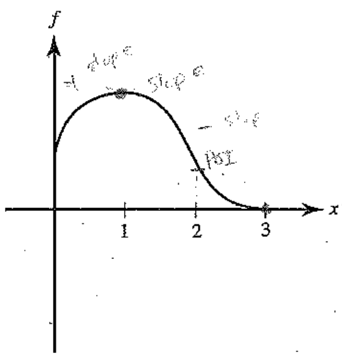


(A)

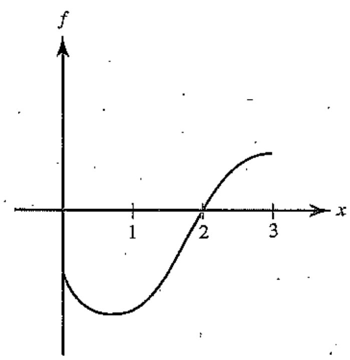


(B)

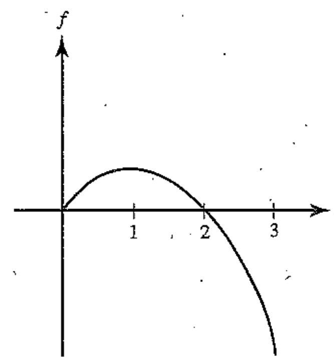




(D)



(E)



Use t

88.

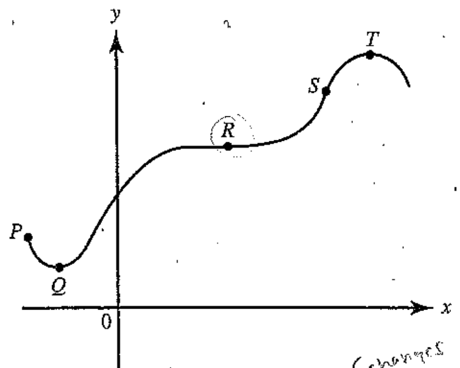
89.

90.

91.

92.

Use the following graph for Questions 88–90.



slope = 0 PoI (changes concavity)

88. At which labeled point do both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ equal zero?

(A) P (B) Q (C) R (D) S (E) T

89. At which labeled point is $\frac{dy}{dx}$ (slope) positive and $\frac{d^2y}{dx^2}$ (PoI) equal to zero? goes from concave up + CCD

(A) P (B) Q (C) R (D) S (E) T

90. At which labeled point is $\frac{dy}{dx}$ equal to (Slope) zero and $\frac{d^2y}{dx^2}$ (CCD) negative?

(A) P (B) Q (C) R (D) S (E) T

91. If $f(6) = 30$ and $f'(x) = \frac{x^2}{x+3}$, estimate $f(6.02)$ using the line tangent to f at $x = 6$.

(A) 29.92 (B) 30.02 (C) 30.08
(D) 34.00 (E) none of these

$f'(6) = \frac{6^2}{6+3} = 4$

→ pt $(6,30)$

$y - 30 = 4(x-6)$

$y = 4(x-6) + 30$

$f(x) = 4(x-6) + 30$

$f(6.02) = 4(6.02-6) + 30$

$= 4(0.02) + 30$

$= 30.08$

92. The local linear approximation for $f(x) = \sqrt{x^2+16}$ near $x = -3$ is

(A) $5 - \frac{3}{5}(x-3)$ (B) $5 + \frac{3}{5}(x-3)$ (C) $5 - \frac{3}{5}(x+3)$

(D) $3 - \frac{5}{3}(x-3)$ (E) $3 + \frac{3}{5}(x+3)$

★ for 92 $f'(x) =$ ____

$f'(-3) =$ ____

Answer Key

1.	D	**21.**	E	**41.**	E	**61.**	A	**81.**	E
2.	A	**22.**	B	**42.**	D	**62.**	C	**82.**	D
3.	E	**23.**	D	**43.**	E	**63.**	B	**83.**	B
4.	B	**24.**	A	**44.**	D	**64.**	E	**84.**	B
5.	D	**25.**	D	**45.**	C	**65.**	C	**85.**	B
6.	C	**26.**	C	**46.**	B	**66.**	E	**86.**	D
7.	D	**27.**	E	**47.**	A	**67.**	D	**87.**	C
8.	D	**28.**	A	**48.**	D	**68.**	C	**88.**	C
9.	C	**29.**	B	**49.**	E	**69.**	E	**89.**	D
10.	B	**30.**	D	**50.**	E	**70.**	B	**90.**	E
11.	D	**31.**	C	**51.**	A	**71.**	A	**91.**	C
12.	E	**32.**	B	**52.**	E	**72.**	C	**92.**	C
13.	C	**33.**	D	**53.**	C	**73.**	D		
14.	A	**34.**	A	**54.**	E	**74.**	E		
15.	B	**35.**	B	**55.**	B	**75.**	B		
16.	E	**36.**	C	**56.**	D	**76.**	B		
17.	B	**37.**	E	**57.**	D	**77.**	B		
18.	B	**38.**	C	**58.**	D	**78.**	D		
19.	D	**39.**	A	**59.**	E	**79.**	D		
20.	A	**40.**	A	**60.**	D	**80.**	C		

Answers Explained

1. (D) Substituting $y = 2$ yields $x = 1$. We find y' implicitly.

$$3y^2y' - (2xyy' + y^2) = 0; \quad (3y^2 - 2xy)y' - y^2 = 0.$$

Replace x by 1 and y by 2; solve for y'.

2. (A) $2yy' - (xy' + y) - 3 = 0$. Replace x by 0 and y by -1; solve for y'.

3. (E) Find the slope of the curve at $x = \frac{\pi}{2}$: $y' = x\cos x + \sin x$. At $x = \frac{\pi}{2}$,

$y' = \frac{\pi}{2} \cdot 0 + 1$. The equation is $y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right)$.

4. (B) Since $y' = e^{-x}(1 - x)$ and $e^{-x} > 0$ for all x, $y' = 0$ when $x = 1$.

5. (D) The slope $y' = 5x^4 + 3x^2 - 2$. Let $g = y'$. Since $g'(x) = 20x^3 + 6x = 2x(10x^2 + 3)$, $g'(x) = 0$ only if $x = 0$. Since $g''(x) = 60x^2 + 6$, g'' is always positive, assuring that $x = 0$ yields the minimum slope. Find y' when $x = 0$.

6. (C) Since $2x - 2yy' = 0$, $y' = \frac{x}{y}$. At (4, 2), $y' = 2$. The equation of the tangent at (4, 2) is $y - 2 = 2(x - 4)$.

7. (D) Since $y' = \frac{y}{2y - x}$, the tangent is vertical for $x = 2y$. Substitute in the given equation and solve for y.

8. (D) Since $V = \frac{4}{3}\pi r^3$, therefore, $dV = 4\pi r^2 dr$. The approximate increase in volume is $dV \approx 4\pi(3^2)(0.1)$ in^3.